Topics in Perturbative QCD

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- New Method for Computing NLO Quarkonium Rates to All Orders in v
- Factorization Theorems for QCD
  - Factorization Theorems for Exclusive Quarkonium Production
  - Closing a Loop-Hole in Proofs of Factorization Theorems

New Method for Computing NLO Quarkonium Rates to All Orders in v

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- Computation of quarkonium decay and production rates requires matching of amplitudes between full QCD and Nonrelativistic QCD (NRQCD).
- At one-loop level, the matching calculation at all orders in the heavy-quark velocity v is daunting.
  - Requires operators and coefficients of all orders in v.
  - Requires one-loop renormalizations of operators by interactions of all orders in v.
- Instead, compute the NRQCD part of the matching by starting with full QCD.
  - Carry out integrations over the time component of the loop momentum by the contour method.
  - Expand integrands in powers of momenta divided by m_c.
  - Discard scaleless integrals in dimensional regularization.
  - Sum the resulting expression to all orders in v.
- Application to the heavy-quark electromagnetic current
  - Important for decay and production of quarkonium through a virtual photon (e^+e^- colliders).

- The resulting one-loop expression for the EM current to all orders in v is very compact:

$$i\mathcal{A}^{i}_{Q\bar{Q}_1} = \bar{v}(p_2)(G\gamma^{i} + Hq^{i})u(p_1),$$

$$\Delta G^{(1)}_{\overline{\mathrm{MS}}} = \frac{\alpha_s C_F}{4\pi}\left\{2\left[(1+\delta^2)L(\delta)-1\right]\log\frac{\mu^2}{m^2} + 6\delta^2 L(\delta) - 4(1+\delta^2)K(\delta) - 4\right\},$$

$$\Delta H^{(1)} = \frac{\alpha_s C_F}{4\pi}\frac{2(1-\delta^2)}{m}L(\delta).$$

$$L(\delta) = \frac{1}{2\delta}\log\left(\frac{1+\delta}{1-\delta}\right),$$

$$K(\delta) = \frac{1}{4\delta}\left[\mathrm{Sp}\left(\frac{2\delta}{1+\delta}\right) - \mathrm{Sp}\left(-\frac{2\delta}{1-\delta}\right)\right],$$

$$\delta = \frac{v}{\sqrt{1+v^2}}.$$

- The method recovers all of the known fixed-order results.
- The labor involved in the new approach in calculating to all orders in v is comparable to the labor involved in traditional approaches in calculating to order v^2.

- The v expansion converges very quickly:

$$\sum_{n=0}^{0} \left[s_n^{(1)}\right]_{\overline{\rm MS}} [\langle \boldsymbol{q}^2 \rangle_{J/\psi}]_{\overline{\rm MS}}^n = -\frac{\alpha_s C_F}{4\pi} \times 8,$$

$$\sum_{n=0}^{1} \left[s_n^{(1)}\right]_{\overline{\rm MS}} [\langle \boldsymbol{q}^2 \rangle_{J/\psi}]_{\overline{\rm MS}}^n = -\frac{\alpha_s C_F}{4\pi} \times 7.826,$$

$$\sum_{n=0}^{2} \left[s_n^{(1)}\right]_{\overline{\rm MS}} [\langle \boldsymbol{q}^2 \rangle_{J/\psi}]_{\overline{\rm MS}}^n = -\frac{\alpha_s C_F}{4\pi} \times 7.883,$$

$$\sum_{n=0}^{3} \left[s_n^{(1)}\right]_{\overline{\rm MS}} [\langle \boldsymbol{q}^2 \rangle_{J/\psi}]_{\overline{\rm MS}}^n = -\frac{\alpha_s C_F}{4\pi} \times 7.872,$$

$$\sum_{n=0}^{\infty} \left[s_n^{(1)}\right]_{\overline{\rm MS}} [\langle \boldsymbol{q}^2 \rangle_{J/\psi}]_{\overline{\rm MS}}^n = -\frac{\alpha_s C_F}{4\pi} \times 7.873.$$

Factorization Theorems for QCD

- Factorization theorems are the cornerstone of all perturbative calculations of hard-scattering cross sections and decay rates in QCD.
  - They separate the high-momentum, perturbative partonic processes from the low-momentum, nonperturbative processes.
  - Predictive power: The nonperturbative physics is parametrized in terms of universal quantities, such as parton distributions, Isgur-Wise functions, NRQCD matrix elements.

Factorization Theorems for Exclusive Quarkonium Production

G.T. Bodwin (ANL), J. Lee (Korea U.), X. Garcia i Tormo (ANL)
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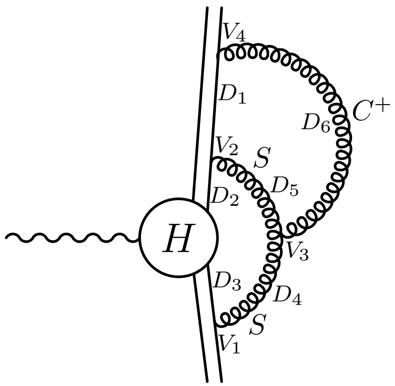
- Using graphical methods, we established factorization theorems for
  - $e^+e^- \to$ charmonium $+$ charmonium,
  - $B \to$ light meson $+$ charmonium.
- The theorems hold to all orders in α_s.
- There are power-suppressed corrections of order
  - $(m_c v^2)^2/s$ for e^+e^- annihilation to two S-wave charmonium,
  - $m_c v^2/m_b$ for B-meson decays to an S-wave charmonium.
- These are the first factorization theorems to be proven for quarkonium production.

Closing a Loop-Hole in Proofs of Factorization Theorems

G.T. Bodwin (ANL), J. Lee (Korea U.), X. Garcia i Tormo (ANL)
arXiv:0903.0569 [hep-ph]

- Existing proofs of factorization theorems, both traditional graphical proofs and proofs in soft-collinear effective theory (SCET), overlook a crucial fact of QCD:
  Low-energy collinear gluons can couple to soft gluons.

- This situation first arises at NNLO in perturbation theory and is relevant to the consistency of calculations at that order.



$$
\begin{aligned}
\text{volume of integration} &\sim Q^8\epsilon_S^4(\epsilon^+)^4(\eta^+)^4\\
V_1\cdot V_2 &\sim Q^2\\
V_3\cdot V_4 &\sim \epsilon_S Q^2\\
D_1 &\sim 1/[Q^2\epsilon^+(\eta^+)^2]\\
D_2 &\sim 1/(Q^2\epsilon_S)\\
D_3 &\sim 1/(Q^2\epsilon_S)\\
D_4 &\sim 1/(Q^2\epsilon_S^2)\\
D_5 &\sim 1/[Q^2(\epsilon_S^2+\epsilon_S\epsilon^+)]\\
D_6 &\sim 1/[Q^2(\epsilon^+)^2(\eta^+)^2]
\end{aligned}
$$

- We pointed out this loop-hole and devised new all-orders methods to deal with it in factorization proofs.

- We demonstrated the new methods by proving to all orders in α_s that the traditional factorization formula holds for $e^+e^- \to$ light meson $+$ light meson.

- This work puts factorization theorems on a more sound theoretical footing.
- The new understanding of the nature of factorization may be useful in removing singularities from calculations in QCD at NNLO and higher.