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# TM - Class Cavity Design

Tutorial

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1. Introduction and History
2. RF Parameters
3. Criteria for Cavity Design
  - a. Accelerating Cavities
  - b. Deflecting Cavities
4. Multi-cell Structures and Weakly Coupled Structures
5. Tools for RF-design
6. LEC and Transient state
7. Mechanical Design
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# 1. Introduction and History

## Milestones that led to accelerators based on SRF

### Superconductivity

1908: Heike Kamerlingh Onnes (Holland)  
Liquefied Helium for the first time.

1911: Heike Kamerlingh Onnes  
Discovered Superconductivity.

1928-34: Walther Meissner (Germany)  
Discovered Superconductivity of Ta, V, Ti and Nb.

### RF Acceleration

1924: Gustaf Ising (Sweden)  
The First Publication on RF Acceleration  
Arkiv för Matematik, Astronomi och Fysik.

1928: Rolf Wideröe (Norway, Germany)  
Built the first RF Accelerator,  
Arch. für Elektrotechnik 21, vol.18.

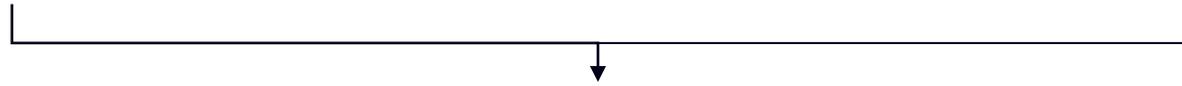
1947: Luis Alvarez (USA)  
Built first DTL (32 MeV protons).

1947: W. Hansen (USA)  
Built first 6 MeV e-accelerator, Mark I  
(TW- structure).

# 1. Introduction and History

Superconductivity

RF Acceleration



**1961: W. Fairbank (Stanford Univ.)**

Presented the first proposal for a superconducting accelerator for electrons

**A. Banford and G. Stafford (Rutherford Appleton Lab.)**

Presented the first proposal for a superconducting accelerator for protons



**1964: W. Fairbank, A. Schwettman, P. Wilson (Stanford Univ.)**

First acceleration of electrons with sc lead cavity



**1970: J. Turneaure (Stanford Univ.)**

$E_{\text{peak}} = 70 \text{ MV/m}$  and  $Q \sim 10^{10}$  in 8.5 GHz cavity !



**1968-1981: M. McAshan, A. Schwettman, T. Smith, J. Turneaure, P. Wilson (Stanford Univ.)**

Developed and Constructed the Superconducting Accelerator SCA

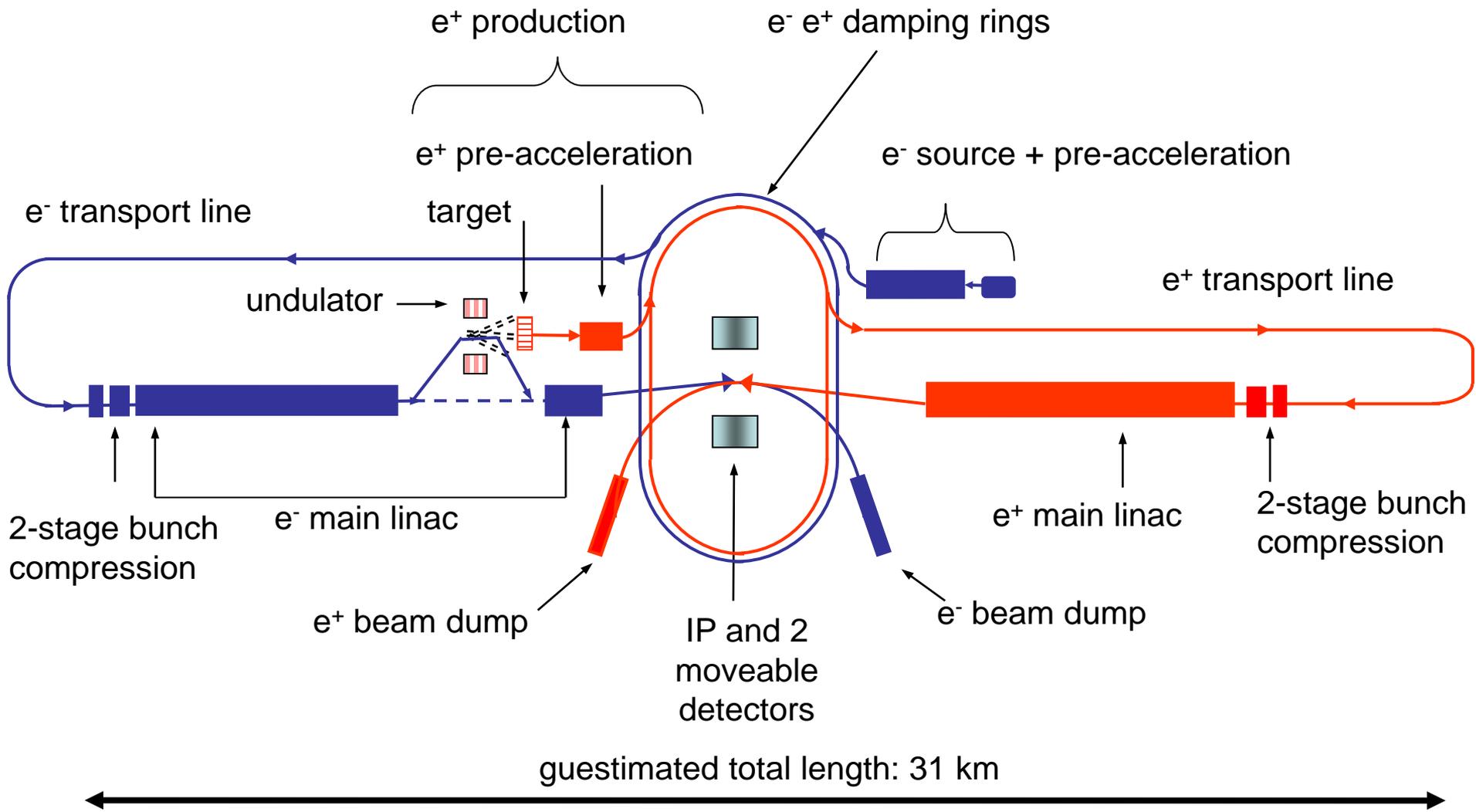
Since then, many sc accelerators were built and we are constructing and making plans for many new facilities.

# 1. Introduction and History

Accelerator	Country	No. of Cavities	SRF Active Length [m]	Status
TRISTAN	Japan	32	49	dismantled
LEP	Switzerland	288	490	dismantled
HERA	Germany	16	19	dismantled
SCA	USA	4	28	shutdown
S-DALINAC	Germany	10	10	operational
CESR	USA	4	1.2	operational
CEBAF	USA	320	160	operational
KEK-B	Japan	8	2.4	operational
Taiwan LS	Taiwan	2	0.6	operational
Canadian LS	Canada	2	0.6	operational
DIAMOND	UK	3	0.9	operational
SOLEIL	France	4	1.7	operational
FLASH	Germany	56	58	operational
SNS	USA	81	65	operational
JLab-FEL	USA	24	14	operational
LHC	Switzerland	16	6	operational
ELBE	Germany	6	6	operational
CEBAF 12GeV	USA	+80	56	construction
European XFEL	Germany	648 (808)	674 (840)	construction
SNS-Upgrade	USA	+36	33	R&D
ERL Cornell	USA	384	310	R&D
RHIC Cooling	USA	1	1	R&D
BEPC II	China	2	0.6	R&D
Project X	USA	Option 352	Option 360	R&D
ILC	Option USA	15764	16395	R&D

# 1. Introduction and History

ILC (~15764/~16395m)



# 1. Introduction and History

The “core elements” of all mentioned facilities were, are or will be sc standing wave accelerating structures.

Photographs of TM-class (elliptical) accelerating structures with  $\beta \geq 0.61$ .

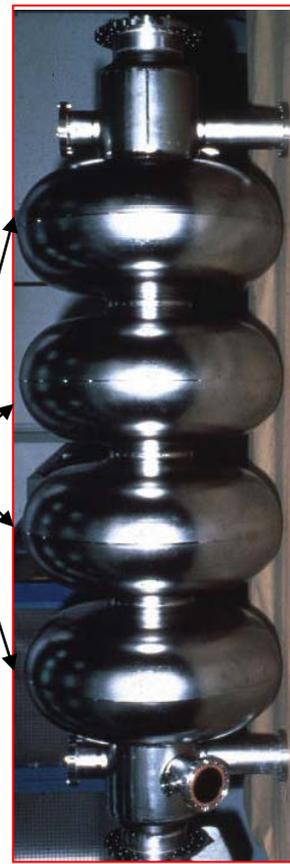
FERMI 3.9 GHz



TESLA/ILC 1.3 GHz



LEP 0.352 GHz



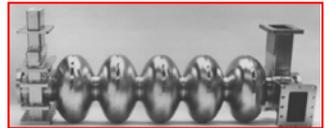
S-DALINAC 3 GHz



SNS  $\beta=0.61, 0.81, 0.805$  GHz



CESR/CEBAF 1.5 GHz



HERA 0.5 GHz



HEPL 1.3 GHz



KEK-B 0.5 GHz



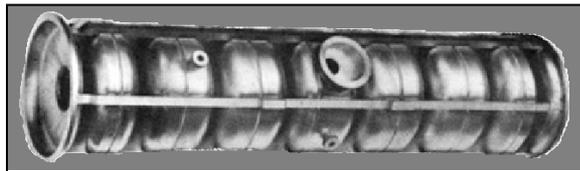
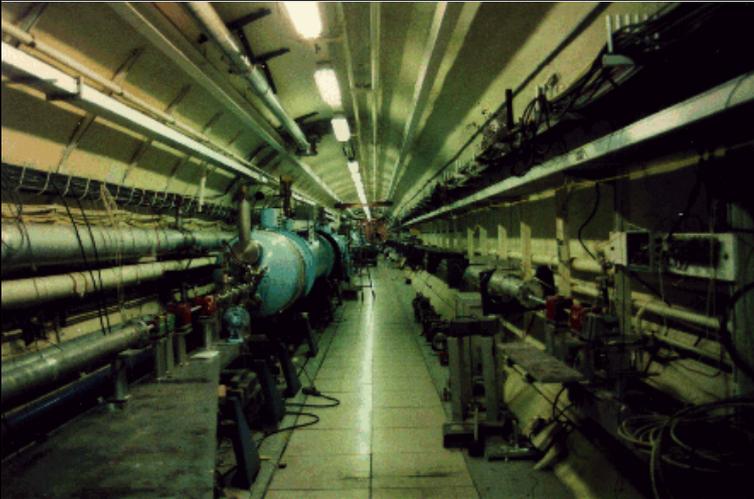
CESR 0.5 GHz



# 1. Introduction and History

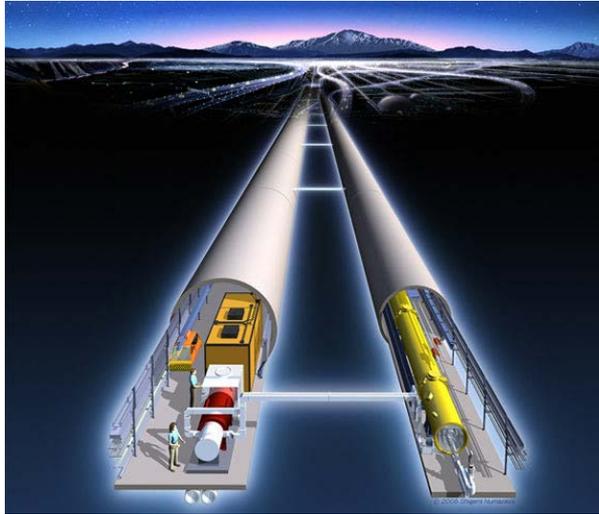
The progress in last 34 years and what do we need in the next 10 years?

~ 28 m long SCA at Stanford, 1977.



$E_{acc} \sim 2$  (2.5) MV/m in cw (10% DF).  
4 Structures 5.65m + capture + pre-accelerator.

~ 21.6 km long ILC linac, 2020+



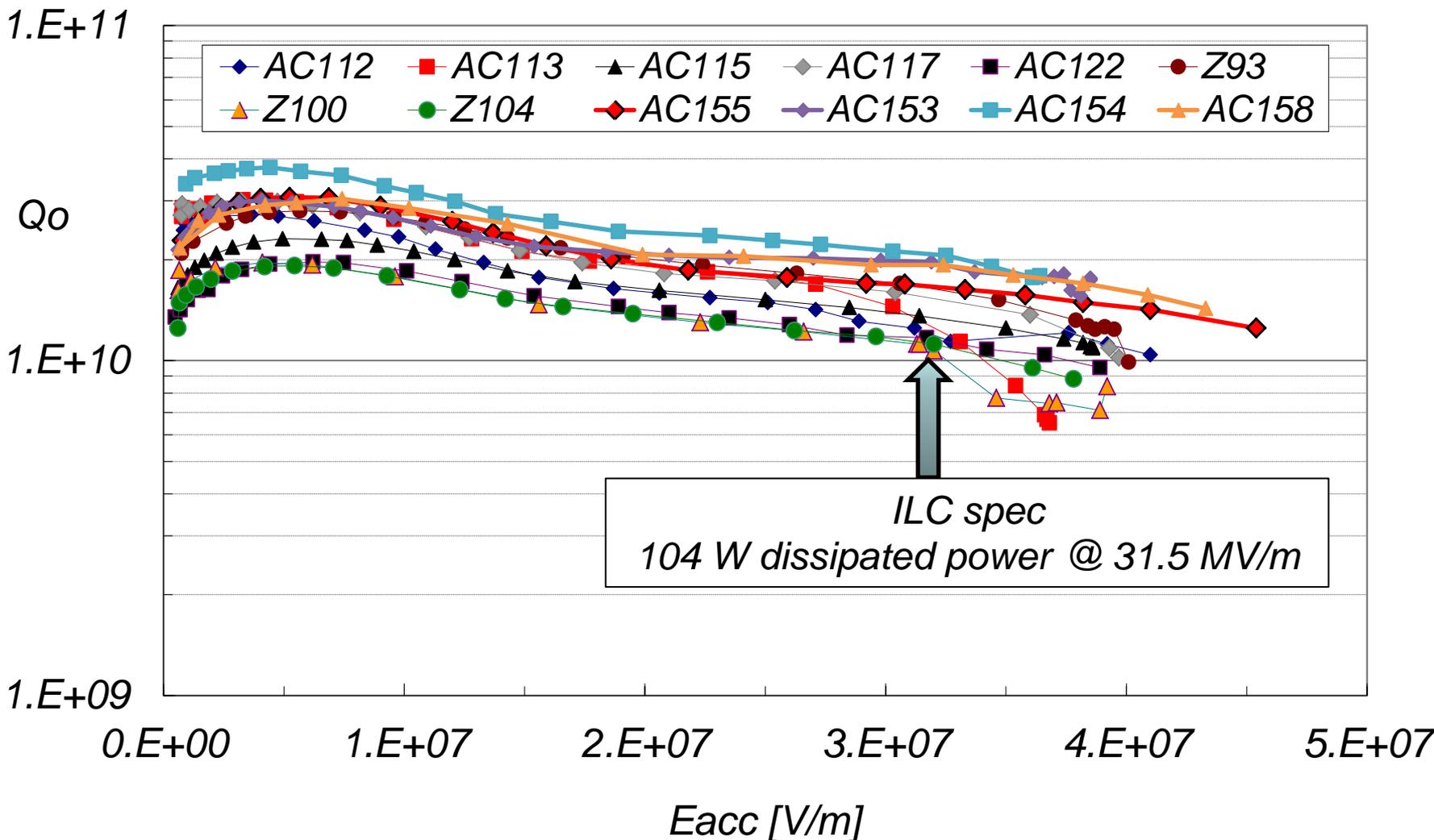
$E_{acc} > 35$  MV/m was demonstrated in many 9-cell cavities in the cw test.



The  $\langle E_{acc} \rangle = 31.5$  MV/m is required in all ILC 15764 cavities.

# 1. Introduction and History

Results at DESY (status July 2011) for 12 electropolished TESLA cavities tested at 2K:



## 2. RF Parameters

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### 2.1. Cavities and their Eigenmodes

Cavity  $\equiv$  a volume, partially closed by metal wall, capable to store the  $E$ - $H$  energy

We will make 3 assumptions, to continue here with a simple example:

1. Stored  $E$ - $H$  fields **are harmonic** in time.
2. **Cylindrical symmetry** (good approximation for body of an accelerating cavity).  
Cylindrical  $(r, \varphi, z)$  coordinates, with  $z$  as the symmetry axis and direction of acceleration

$$\left\{ \begin{array}{l} \nabla_c \times H = i\omega\epsilon E \\ \nabla_c \times E = -i\omega\mu H \\ \nabla_c \cdot E = 0 \\ \nabla_c \cdot H = 0 \end{array} \right.$$

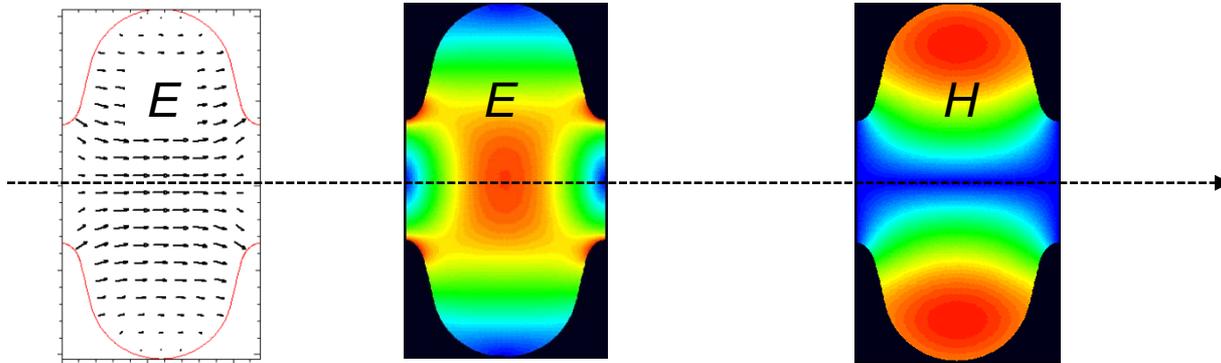
3. Modes suitable for **acceleration have strong  $E$**  along the beam trajectory.  
This enables, by a proper phasing, an efficient energy exchange between a cavity and beam.

## 2. RF Parameters

**TM<sub>0xx</sub>-like monopole** modes have “very strong”  $E_z$  component on the symmetry axis. Their e-m fields do not depend on  $\varphi$ .

$$\frac{\partial \mathbf{E}}{\partial \varphi} = \mathbf{0} \quad \frac{\partial \mathbf{H}}{\partial \varphi} = \mathbf{0}$$

The lowest frequency mode with **TM<sub>010</sub>-like** field pattern is suitable for the acceleration



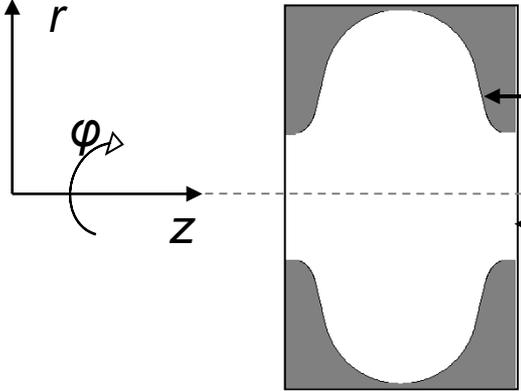
**Non monopole modes (HOM)** have component  $E_z = 0$  on the symmetry axis. Their fields dependent on  $\varphi$ .

TM<sub>110</sub>-like mode is used in crab-cavities for bunch rotation.

## 2. RF Parameters

Maxwell equations + boundary conditions for  $E$  and  $H$  lead to the Helmholtz equation, which is an eigenvalue problem.

For  $H(r,z)$  field of a monopole mode the equation is:



$$(\nabla_c^2 + \omega^2 \varepsilon \mu) H = 0$$

$n \cdot H = 0$  on metal wall  
 $\left\{ \begin{array}{l} H = 0 \\ n \cdot H = 0 \end{array} \right.$  optionally on non metal boundary

$$\nabla_c^2 A = \nabla_c (\nabla_c \cdot A) - \nabla_c \times \nabla_c \times A$$

There is infinity number of TM<sub>0xx</sub> solutions (modes) to the Helmholtz equation.

All modes are determine by:

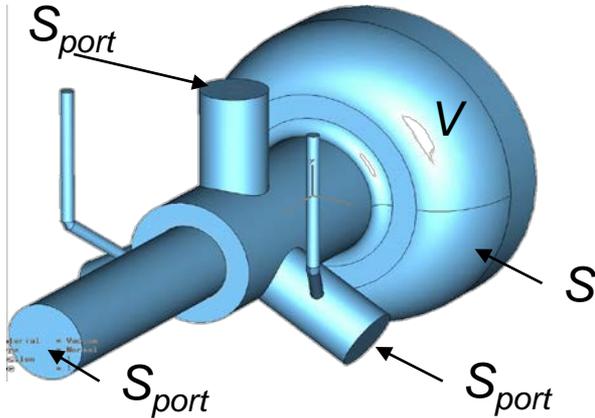
$$H_n(r,z) = [0, H_{\varphi,n}(r,z), 0],$$

$$E_n(r,z) = [E_{r,n}(r,z), 0, E_{z,n}(r,z)]$$

and by frequency  $\omega_n$  ( $n$  is here the index of a mode).

## 2. RF Parameters

### 2.2 What are figures of merit for a cavity storing E-H energy?



$W_n \equiv$  stored energy of a mode  $n : \{\omega_n, E_n, H_n\}$ .

$$W_n \equiv 2\mu \int_V \frac{H_n^2}{4} dV = 2\varepsilon \int_V \frac{E_n^2}{4} dV$$

#### Quality Factors

The measure of the energy loss in metal wall and due to the radiation via open ports:

Intrinsic  $Q \equiv Q_0$

$$Q_{0,n} \equiv \frac{\omega_n \cdot W_n}{P_n} = \frac{\omega_n \cdot W_n}{\frac{R_{s,n}}{2} \int_S H_n^2 ds}$$

External  $Q \equiv Q_{ext}$

$$Q_{ext,n} \equiv \frac{\omega_n \cdot W_n}{P_{rad,n}} = \frac{\omega_n \cdot W_n}{\frac{1}{2} \int_{S_{port}} E_n \times H_n ds}$$

### Geometric Factor

The measure of the energy loss in the metal wall for the surface resistance  $R_{s,n}=1\Omega$

$$G_n \equiv Q_{0,n} \cdot R_{s,n} = \frac{\omega_n \cdot W_n \cdot R_{s,n}}{P_n} = \frac{\omega_n \cdot W_n}{\frac{1}{2} \int_S H_n^2 ds}$$

It is the ratio of the stored energy to integral of  $(H_n)^2$  on the metal wall  $S$ .

## 2.3 What are figures of merit for the beam-cavity interaction?

This interaction which is:

- Acceleration
- Deceleration (ERL)
- HOMs excitation

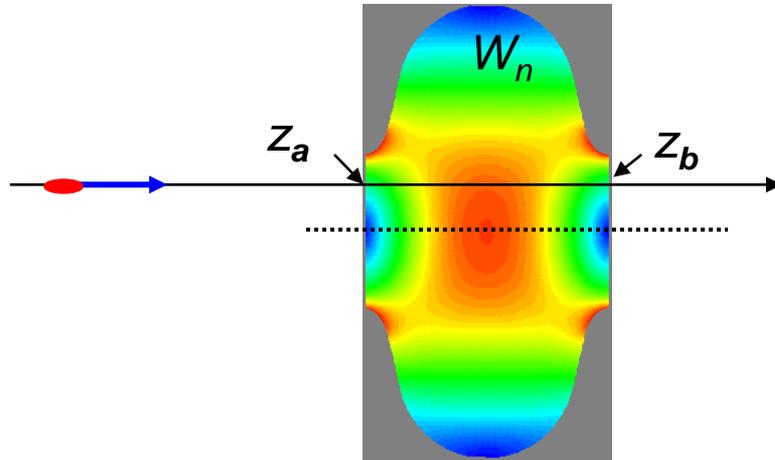
can be described in the Frequency Domain (FD) or/and in Time Domain (TD).

## 2. RF Parameters

$(R/Q)_n$  ; beam impedance

It is a “measure” of the energy exchange between point charge and mode n (FD).

Mode n :  $\{\omega_n, E_n, H_n\}$ .



Trajectory of the point charge q, assumed here is a straight line.

$$V_n = \sqrt{\left( \int_{z_a}^{z_b} E_{n,z} \sin\left(\frac{\omega_n}{\beta c} (z - z_a)\right) dz \right)^2 + \left( \int_{z_a}^{z_b} E_{n,z} \cos\left(\frac{\omega_n}{\beta c} (z - z_a)\right) dz \right)^2}$$

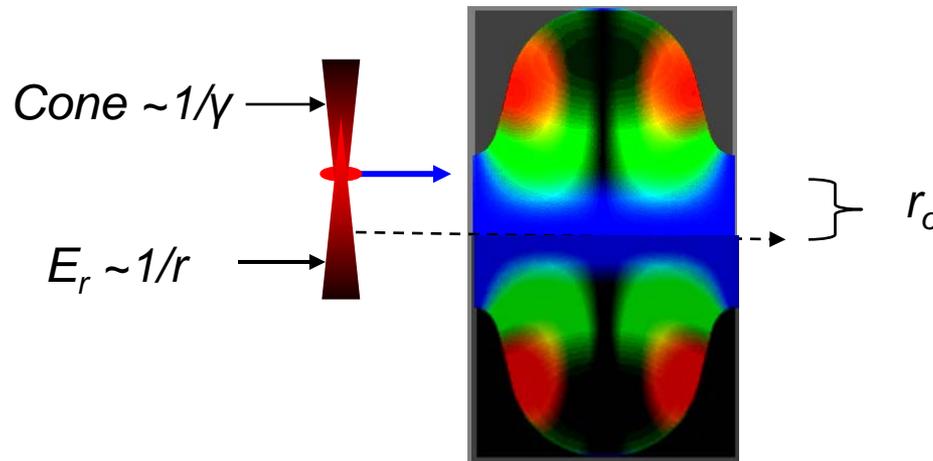
$$(R/Q)_n \equiv \frac{V_n^2}{\omega_n W_n}$$

Linac convention  
for (R/Q) definition.

## 2. RF Parameters

### Longitudinal and Transverse Loss Factors (TD) (Excitation of cavity modes)

Ultra relativistic point charge  $q$  passes empty cavity



- Density of the inducted charge on the wall depends on the distance to beam trajectory
- The non uniform charge density on the metal wall causes the current flow on surface

## 2. RF Parameters

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The amount of energy lost by charge  $q$  to the cavity is:

$$\Delta U_q = k_{\parallel} \cdot q^2 \quad \text{for monopole modes (max. on axis)}$$

$$\Delta U_q = k_{\perp} \cdot q^2 \quad \text{for non monopole modes (off axis)}$$

where  $k_{\parallel}$  and  $k_{\perp}(r)$  are the loss factors for monopole and transverse modes respectively

The induced  $E$ - $H$  field (wake) is a superposition of all cavity eigenmodes having the  $E_n(r, \varphi, z)$  field along the particle trajectory.

Both description methods FD and TD are equivalent.

For an individual mode  $n$  and point-like charge:

$$k_{\parallel, n}^p = \frac{\omega_n \cdot (R / Q)_n}{4}$$

Similar for other loss factors.....

## 2. RF Parameters

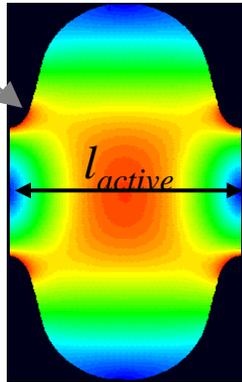
### 2.4. Some “practical” RF parameters for the accelerating mode

For the stored energy  $W_{acc}$  mean value of the accelerating gradient is:

$$E_{acc} = \frac{\sqrt{\omega_{acc} \cdot W_{acc} \cdot (R/Q)_{acc}}}{l_{active}}$$

$E_{peak}$  on the wall

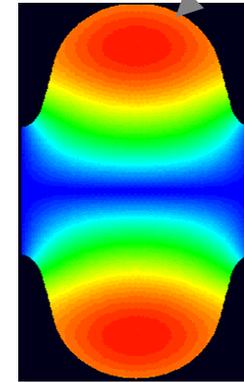
$$\frac{E_{peak}}{E_{acc}}$$



Contour plot of  $|E|$

$B_{peak}$  on the wall

$$\frac{B_{peak}}{E_{acc}}$$



Contour plot of  $|B|$

The ratio shows sensitivity of the shape to the field electron emission phenomenon.

The ratio shows limit in  $E_{acc}$  due to the break-down of superconductivity (Nb ~190 mT).

## 2. RF Parameters

$$G_{acc} \cdot (R/Q)_{acc}$$

For the accelerating mode we often use the product:  $G_{acc} \cdot (R/Q)_{acc}$ , as a “measure” of the power  $P$  dissipated in the wall at given accelerating voltage  $V_{acc}$  and given surface resistance  $R_s$ .

$$\frac{P_{dissipated}}{V_{acc}^2} \equiv \frac{R_s}{G_{acc} \cdot (R/Q)_{acc}}$$

Big improvement is possible:

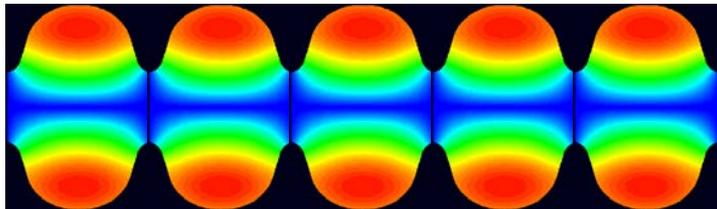
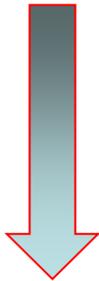
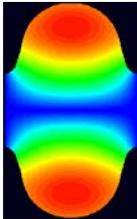
- Due to superconductivity
- Due to the surface quality.

This is due to the geometry of cells  
Moderate improvement possible.

## 2. RF Parameters

cell-to-cell coupling  $k_{cc}$

The  $k_{cc}$  is relevant for the accelerating mode passband of multi-cell structures

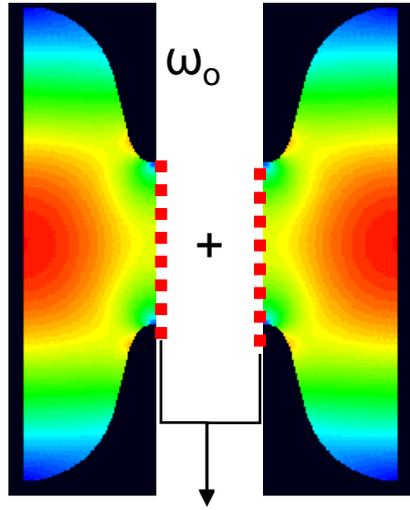


Single-cell structures are attractive because:

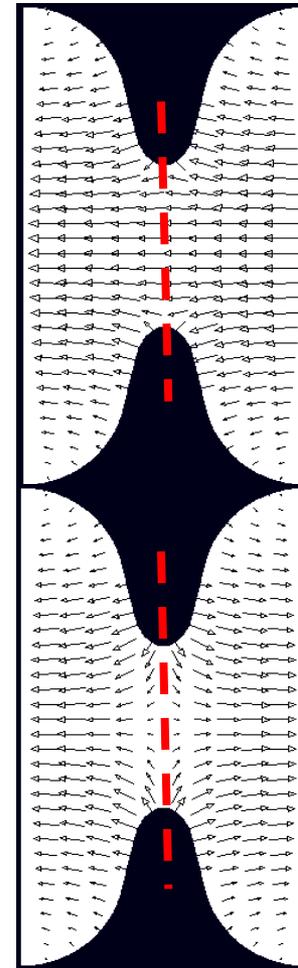
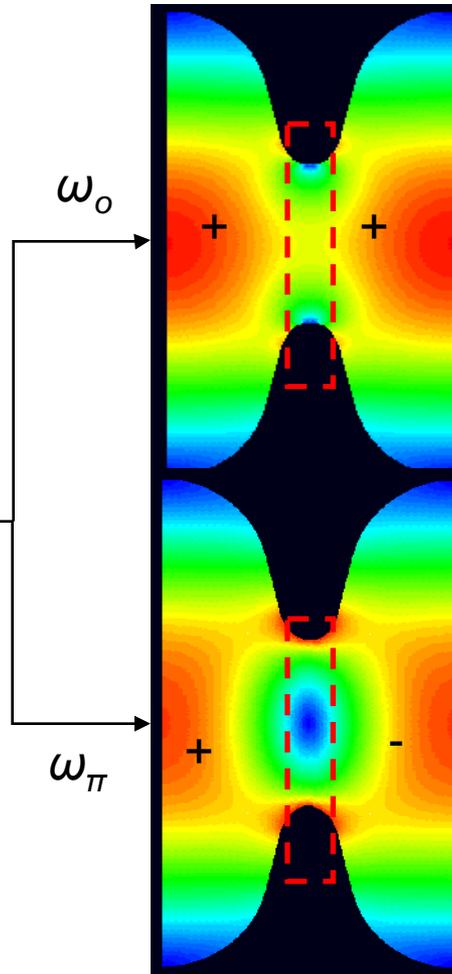
- ◆ It is easier to manage HOM damping
- ◆ There is no field flatness problem.
- ◆ Input coupler transfers less power
- ◆ They are easy for cleaning and preparation
- ◆ **But it is expensive to base even a small linear accelerator on the single cells. We do it only for very high beam current machines.**
- ◆ Multi-cell structures are less expensive/m and allow for higher real-estate gradient.

## 2. RF Parameters

Resonators closed  
by metal wall:



Symmetry planes for  
the H field

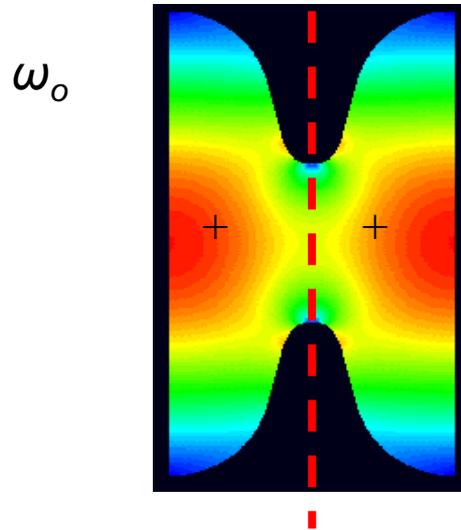


Symmetry plane for  
the H field

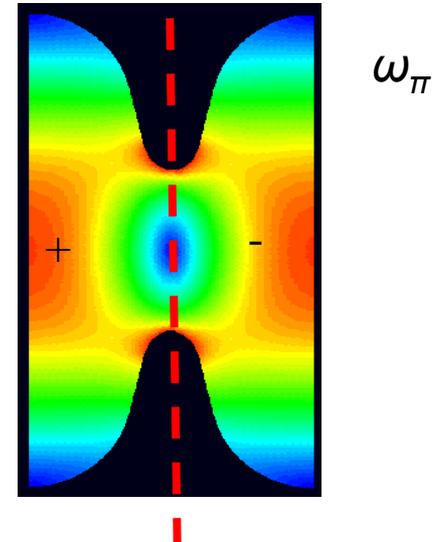
Symmetry plane for  
the E field,  
which is an  
additional solution

## 2. RF Parameters

The energy flux across the coupling region, refills dissipated energy in cells.  
It is proportional to the transverse components:  $H_\phi$  and  $E_r$



Small  $E_r$  (due to the losses) + strong  $H_\phi$  at the symmetry plane

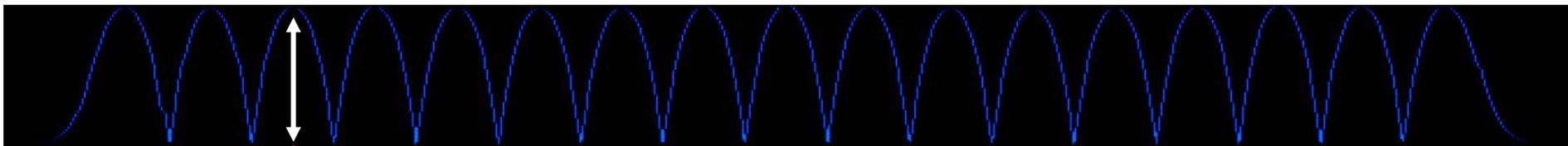
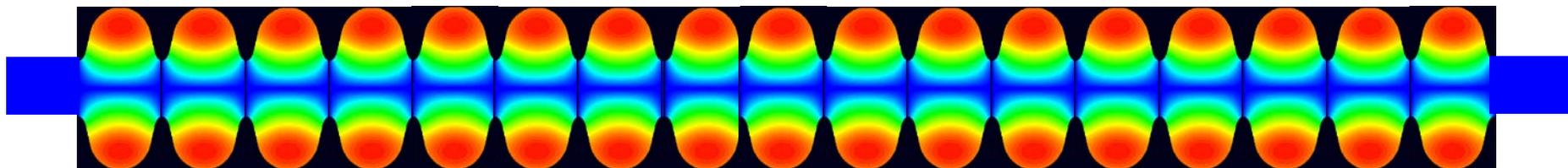


Small  $H_\phi$  (due to the losses) + strong  $E_r$  at the symmetry plane

The normalized difference between  $\omega_0$  and  $\omega_\pi$  is a measure of the energy flow via the coupling region

$$k_{cc} = \frac{\omega_\pi - \omega_0}{\frac{\omega_\pi + \omega_0}{2}}$$

## 2. RF Parameters



$$\frac{\Delta A_j}{A_j} = a_{ff} \frac{\Delta f_j}{f_j}$$

Field flatness factor  $a_{ff}$  for structure made of  $N$  cells and the coupling factor  $k_{cc}$

$$a_{ff} = \frac{N^2}{k_{cc}}$$

The above formulae estimate the sensitivity of a multi-cell field profile to frequency errors of an individual cell for the accelerating mode ( $\pi$ -mode)

### 3. Criteria for Cavity Design; accelerating cells

We will discuss here design of inner cells because they “dominate” the RF properties of multi-cell structures.

RF parameters summary:

$$\text{FM} : (R/Q), G, E_{peak}/E_{acc}, B_{peak}/E_{acc}, k_{cc}$$

$$\text{HOM} : k_{\perp}, k_{\parallel}.$$

The elliptical shape, which we will focus on, used since late 60's, has two major and indispensable features:

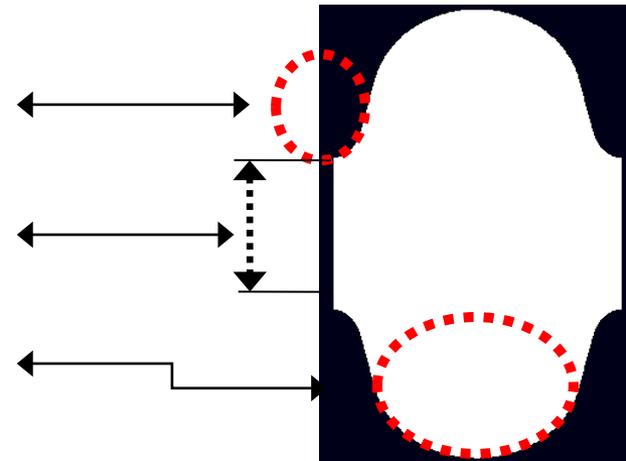
- It is easy to clean
- It demonstrates very little and usually conditionable multipacting.

Geometry :

iris ellipsis : *half-axis*  $h_r, h_z$

iris radius :  $r_i$

equator ellipsis : *half-axis*  $h_r, h_z$



There is some kind of conflict: 7 parameters and only 5 variables to “tune”.

### 3. Criteria for Cavity Design: accelerating cells

<i>Criterion</i>	<i>RF-parameter</i>	<i>Improve(s) when</i>	<i>Cavity examples</i>
<i>Operation at high gradient</i>	$E_{peak}/E_{acc}$ $B_{peak}/E_{acc}$ ↓	$r_i$ ↓ <i>Iris &amp; Equator shape</i>	<i>TESLA,</i> <i>HG CEBAF-12 GeV</i>
<i>Low cryogenic losses</i>	$(R/Q) \cdot G$ ↑	$r_i$ ↓ <i>Equator shape</i>	<i>LL CEBAF-12 GeV</i>
<i>High <math>I_{beam} \leftrightarrow</math></i> <i>Low HOM impedance</i>	$k_{\perp}, k_{\parallel}$ ↓	$r_i$ ↑	<i>B-Factory</i> <i>RHIC cooling</i>

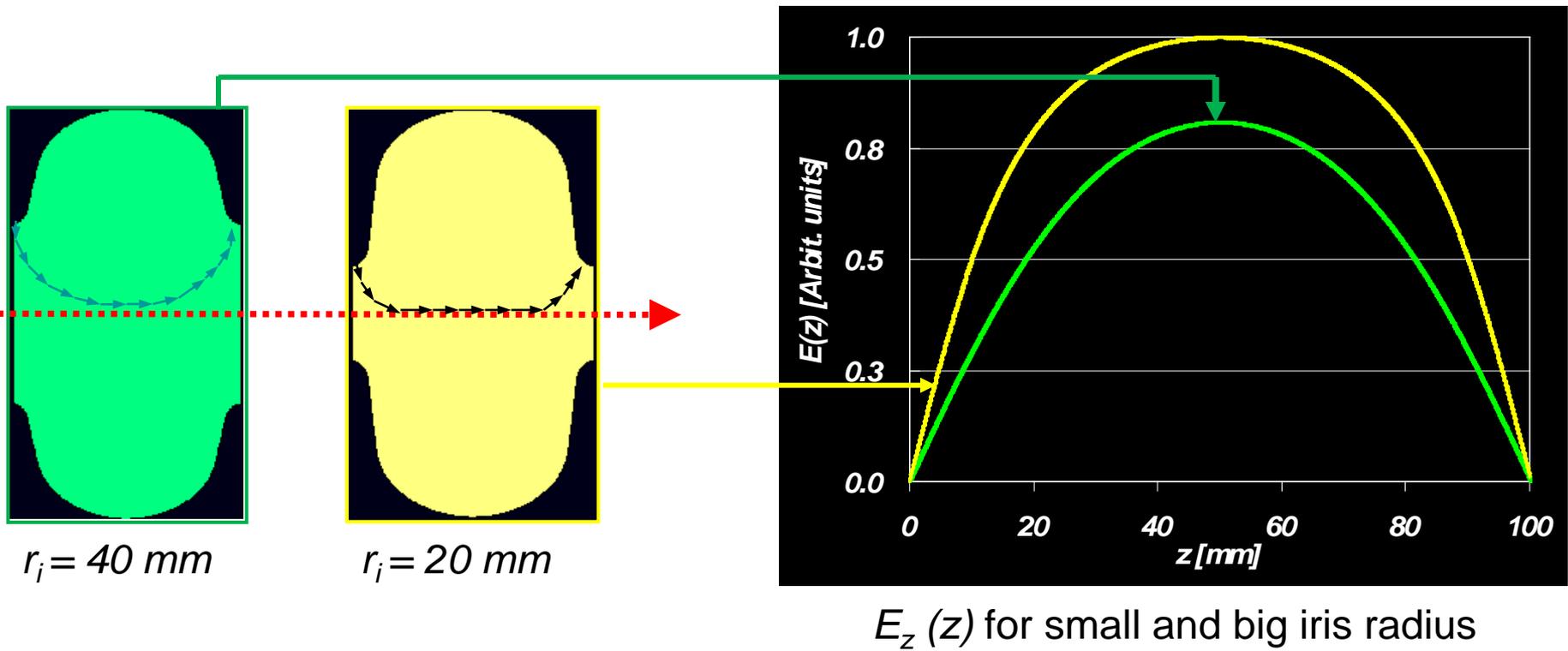
We see here that  $r_i$  is a very “powerful variable” to trim the RF-parameters of a cavity.

### 3. Criteria for Cavity Design: accelerating cells

Why for a smaller aperture ( $r_i$ )

- $(R/Q)$  is bigger
- $E_{peak}/E_{acc}$ ,  $B_{peak}/E_{acc}$  are lower ?

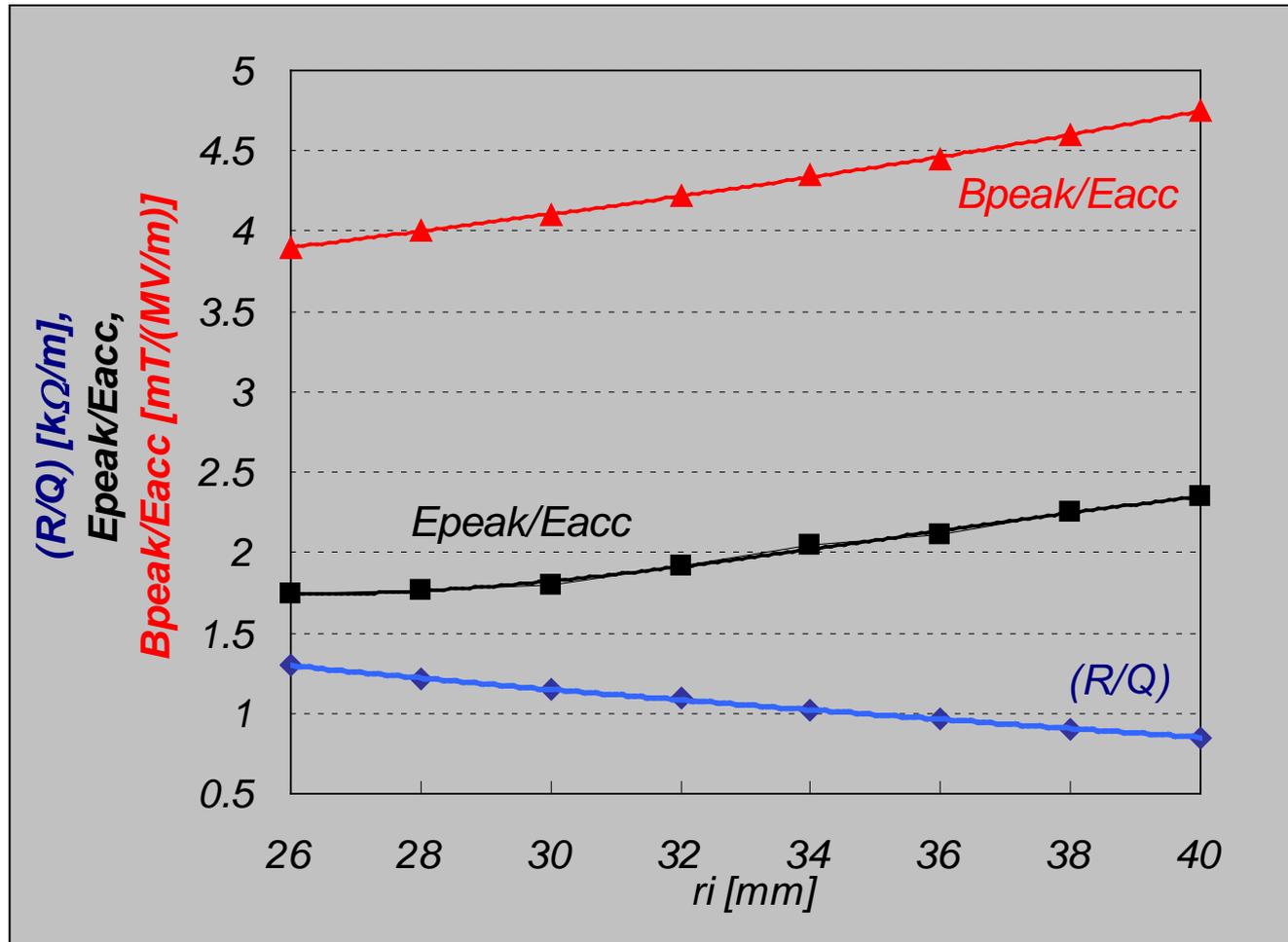
$E_{acc}$  is higher at the same stored energy in the cell with smaller aperture.



### 3. Criteria for Cavity Design: accelerating cells

Example:

$\{ (R/Q), E_{peak}/E_{acc}, B_{peak}/E_{acc} \}$  vs.  $r_i$  for cell at  $f = 1.5$  GHz



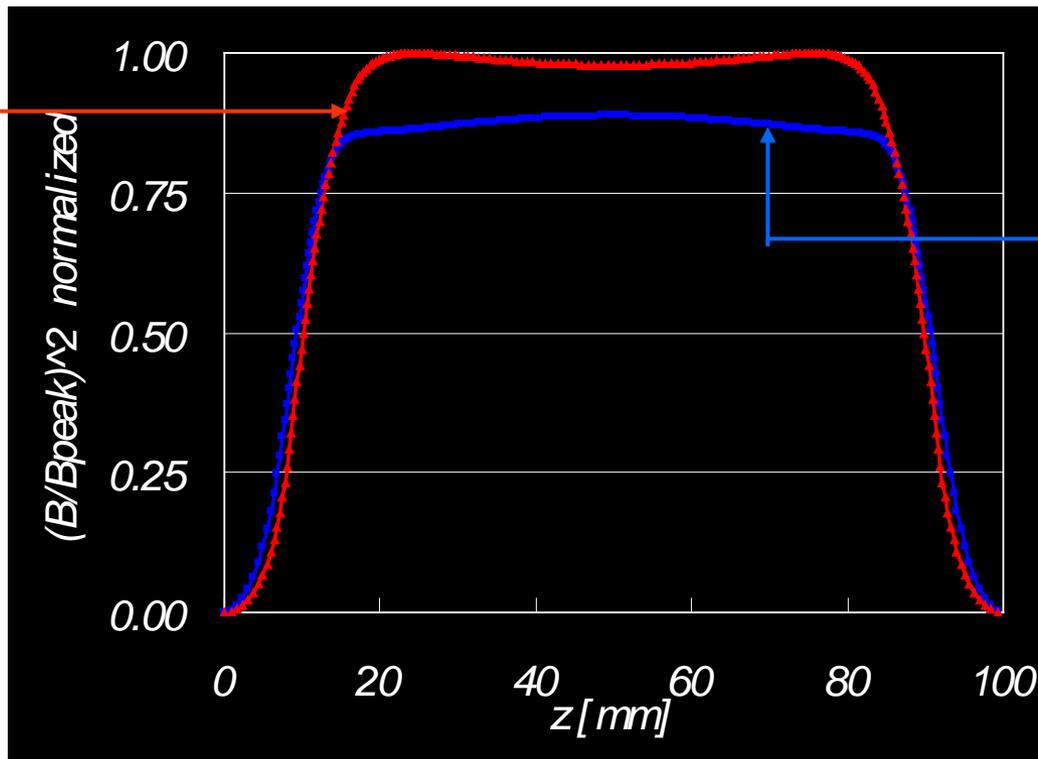
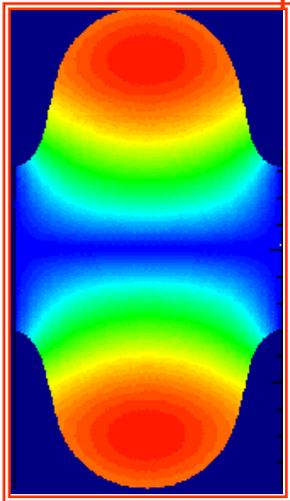
A. Mosnier, E. Haeberl, SRF Workshop 1991

### 3. Criteria for Cavity Design: accelerating cells

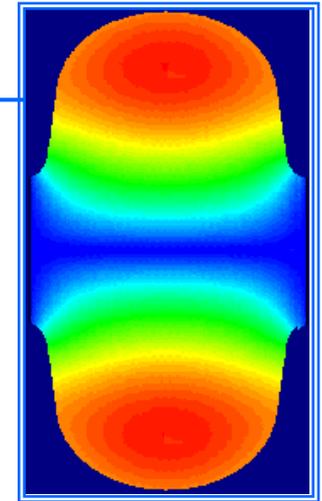
In addition to the iris radius  $r_i$ :

- $B_{peak}/E_{acc}$  (and  $G$ ) changes vs. the equator shape

1 Joule

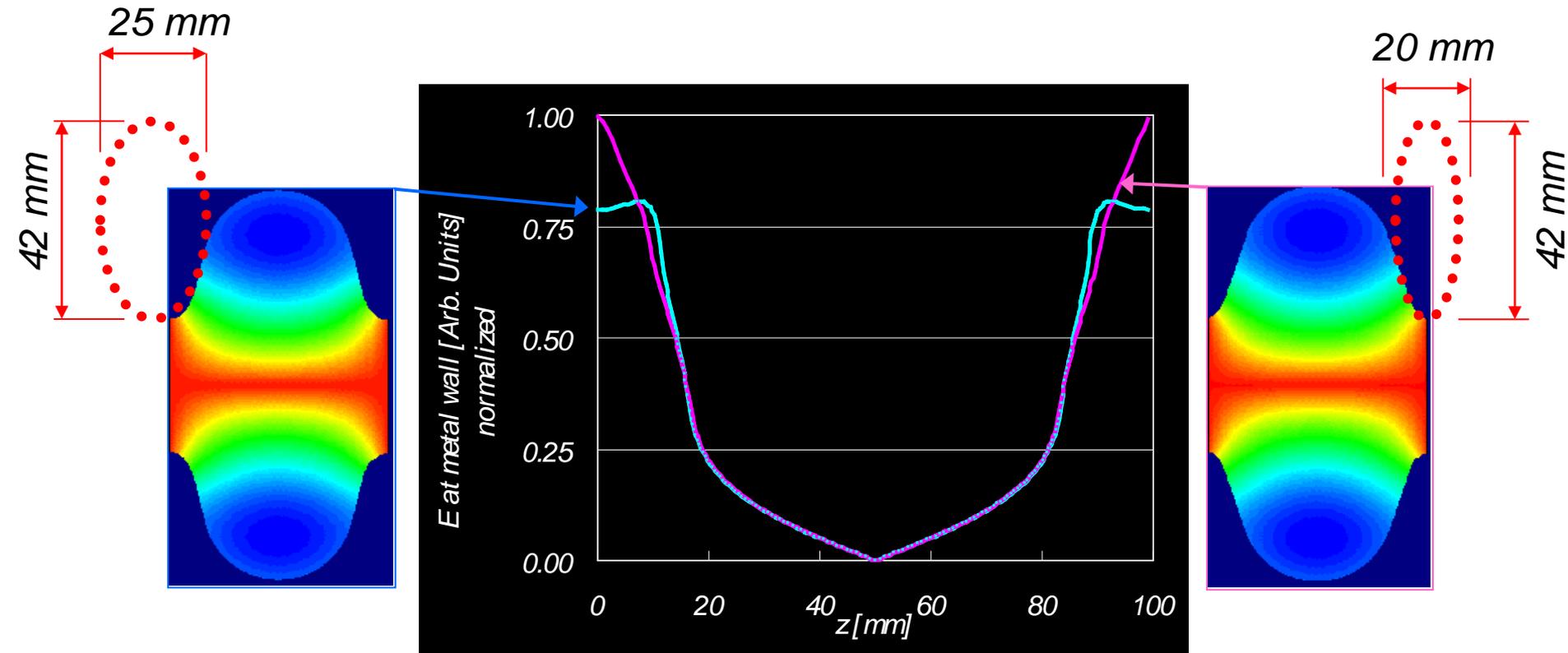


1 Joule



### 3. Criteria for Cavity Design: accelerating cells

Similarly :  $E_{peak}/E_{acc}$  changes vs. the iris shape



Both cells have the same:  $f$ ,  $(R/Q)$  and  $r_i$

### 3. Criteria for Cavity Design: accelerating cells

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We know that a smaller aperture  $r_i$  makes FM :

- $(R/Q)$  higher
- $B_{peak}/E_{acc}$  ,  $E_{peak}/E_{acc}$  lower

} (+)

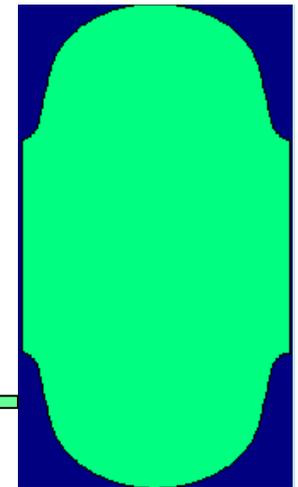
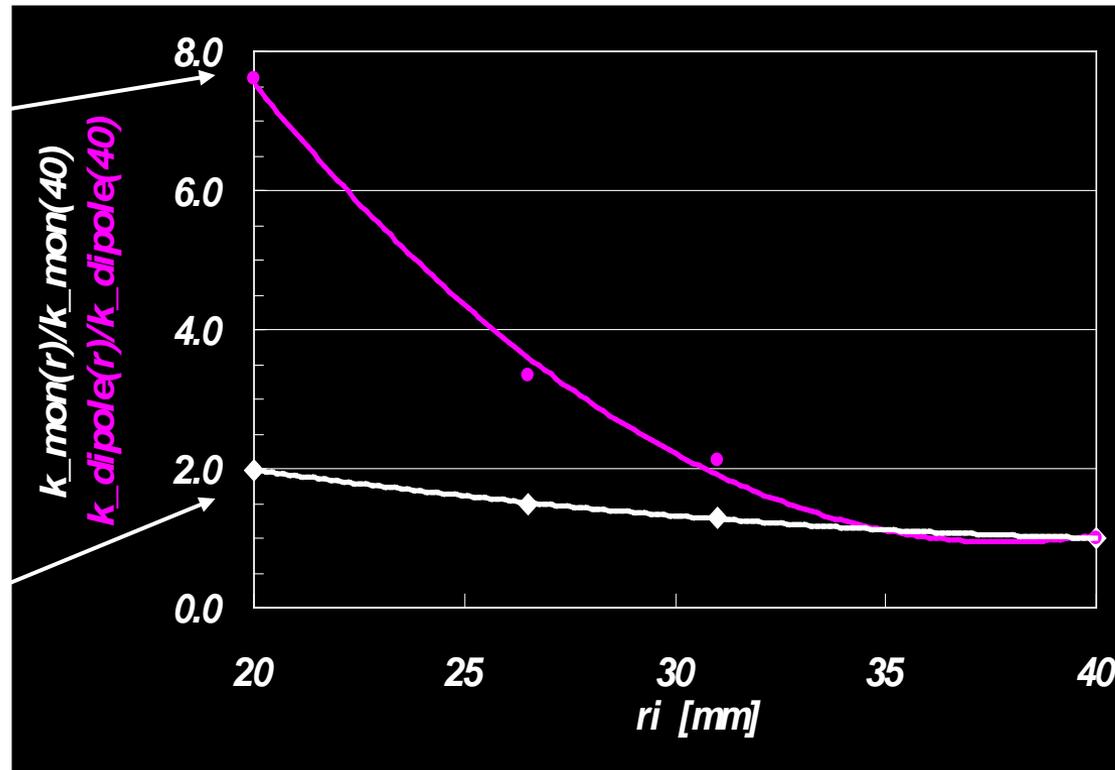
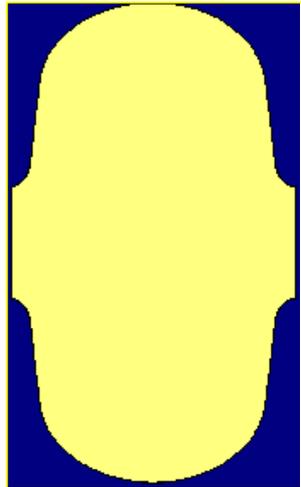
but unfortunately a smaller aperture  $r_i$  makes:

- HOMs impedances  $k_{\perp}$  and  $k_{\parallel}$  higher
- cell-to-cell coupling  $k_{cc}$  weaker

} (-)

### 3. Criteria for Cavity Design: accelerating cells

HOMs loss factors ( $k_{\perp}$ ,  $k_{\parallel}$ )



$$(R/Q) = 152 \Omega$$

$$B_{peak}/E_{acc} = 3.5 \text{ mT}/(\text{MV}/\text{m})$$

$$E_{peak}/E_{acc} = 1.9$$

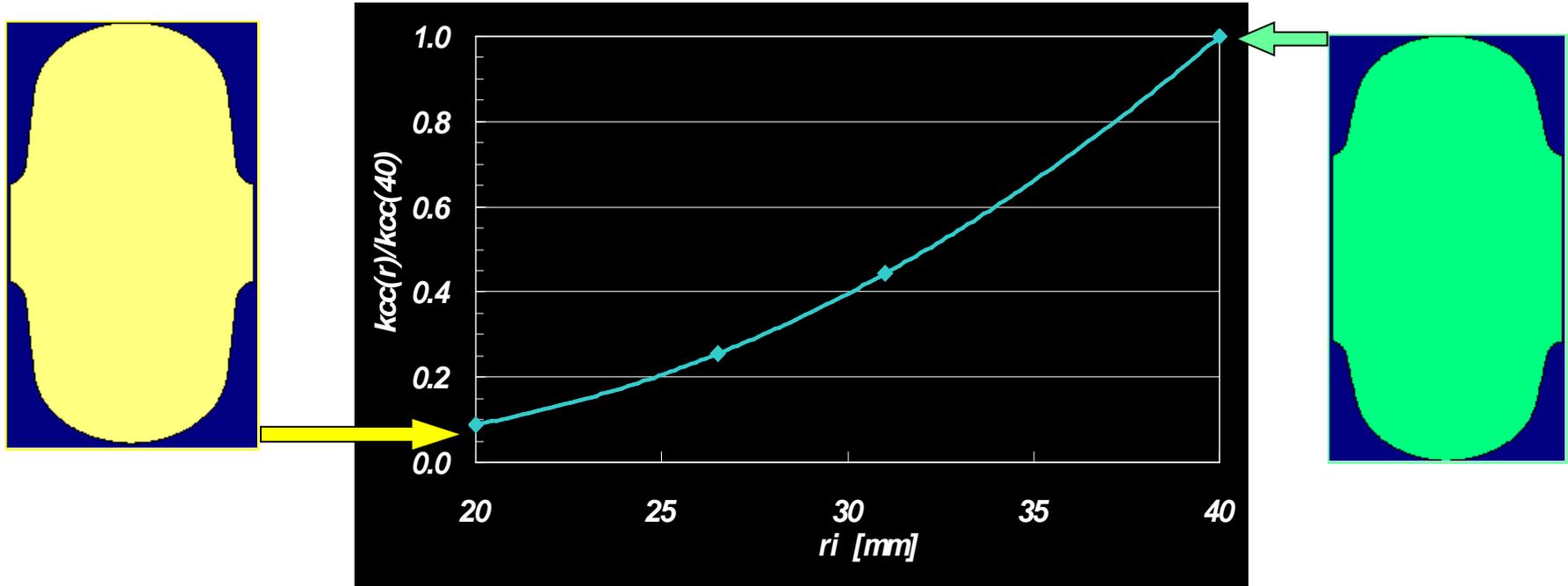
$$(R/Q) = 86 \Omega$$

$$B_{peak}/E_{acc} = 4.6 \text{ mT}/(\text{MV}/\text{m})$$

$$E_{peak}/E_{acc} = 3.2$$

### 3. Criteria for Cavity Design: accelerating cells

Cell-to-cell coupling,  $k_{cc}$



$$(R/Q) = 152 \Omega$$

$$B_{peak}/E_{acc} = 3.5 \text{ mT}/(\text{MV}/\text{m})$$

$$E_{peak}/E_{acc} = 1.9$$

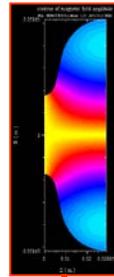
$$(R/Q) = 86 \Omega$$

$$B_{peak}/E_{acc} = 4.6 \text{ mT}/(\text{MV}/\text{m})$$

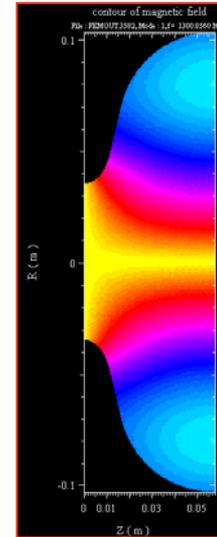
$$E_{peak}/E_{acc} = 3.2$$

### 3. Criteria for Cavity Design: accelerating cells

Frequency of the accelerating mode



$\times 2 =$



$f_{\pi}$	[MHz]	2600
$R/Q$	[ $\Omega$ ]	57
$r/q=(R/Q)/l$	[ $\Omega/m$ ]	2000
$G$	[ $\Omega$ ]	271

$f_{\pi}$	[MHz]	1300
$R/Q$	[ $\Omega$ ]	57
$r/q=(R/Q)/l$	[ $\Omega/m$ ]	1000
$G$	[ $\Omega$ ]	271

$$r/q=(R/Q)/l \sim f$$

### 3. Criteria for Cavity Design: accelerating cells

---

From the formula, we learned before, one obtains:

$$P_{dissipated} = \frac{R_s \cdot V_{acc}^2}{G_{acc} \cdot (r/q)_{acc} \cdot I_{active}} = \frac{R_s \cdot E_{acc}^2 \cdot I_{active}}{G_{acc} \cdot (r/q)_{acc}}$$

Higher  $f_{\pi}$  would be a good choice to minimize dissipation in the metal wall when the length  $l_{active}$  and final energy  $V_{acc}$  are fixed.

Unfortunately this applies only to room temperature structures made of Cu, which

$$R_s(f) \sim (f)^{1/2}.$$

For superconductors, like Nb:

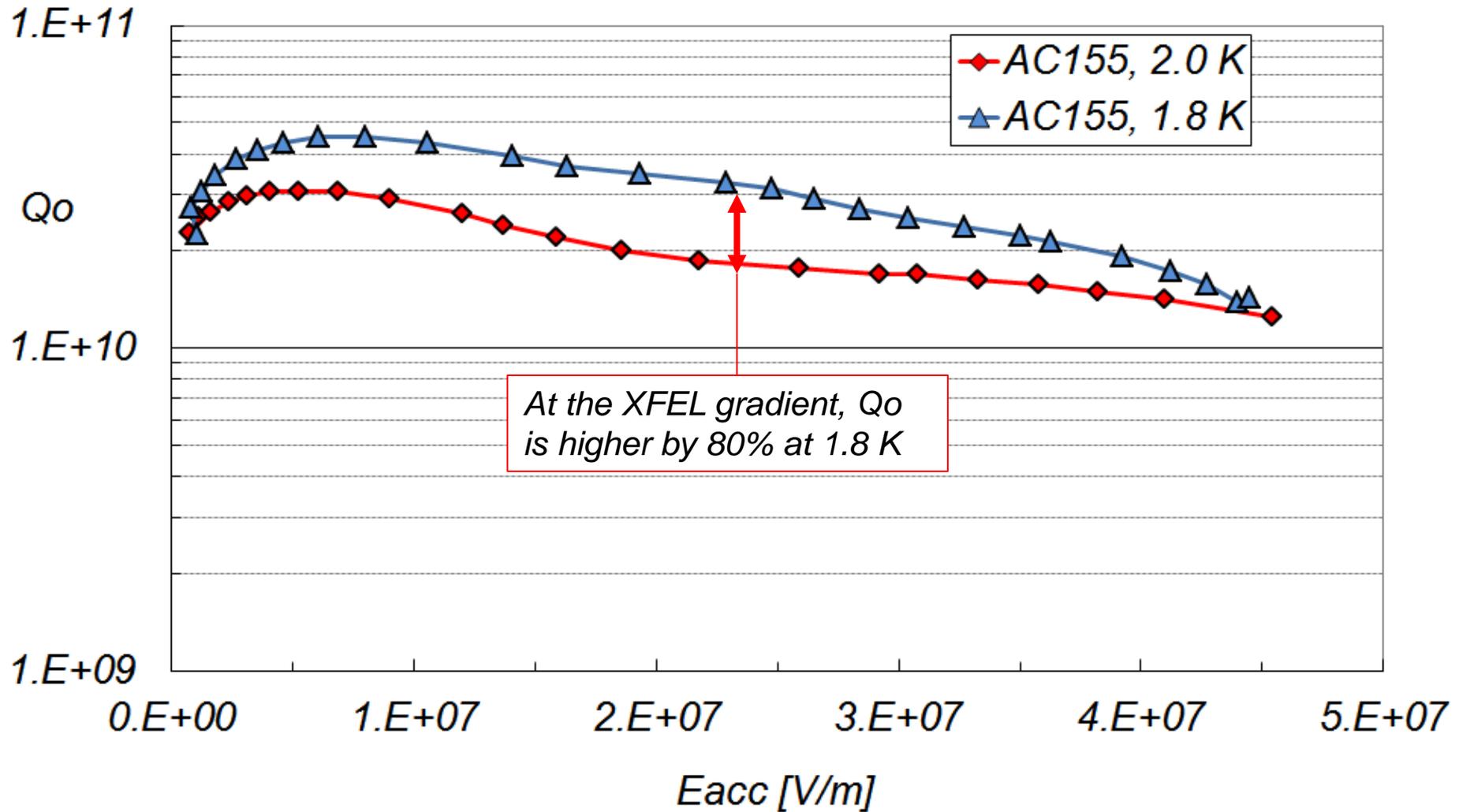
$$R_s(f) = R_{res} + R_{BCS} = R_{res} + 0.0002 \cdot \frac{1}{T} \cdot \left(\frac{f[\text{GHz}]}{1.5}\right)^2 \cdot \exp\left(-\frac{17.67}{T}\right)$$

and  $R_s$ , which is  $\sim (f)^2$  for higher  $f$  must be compensated with lower temperature  $T$ .

This is why ILC, XFEL, ERL (1.3GHz) will operate at 2K (1.8K), and HERA (0.5 GHz) and LEP (0.352 GHz) could operate at 4.2 K.

### 3. Criteria for Cavity Design: accelerating cells

The operation temperature plays a role, especially for high purity Nb:



### 3. Criteria for Cavity Design: accelerating cells

#### Examples of inner cells

		<i>CEBAF Original Cornell <math>\beta=1</math></i>	<i>CEBAF -12 Low Loss <math>\beta=1</math></i>	<i>TESLA <math>\beta=1</math></i>	<i>RHIC Cooler <math>\beta=1</math></i>
$f_{\pi}$	[MHz]	1497.0	1497.0	1300.0	703.7
$k_{cc}$	[%]	3.29	1.49	1.9	2.94
$E_{peak}/E_{acc}$	-	2.56	2.17	1.98	1.98
$B_{peak}/E_{acc}$	[mT/(MV/m)]	4.56	3.74	4.15	5.78
$R/Q$	[ $\Omega$ ]	96.5	128.8	113.8	80.2
$G$	[ $\Omega$ ]	273.8	280	271	225
$R/Q * G$	[ $\Omega * \Omega$ ]	26421	36064	30840	18045
$k_{\perp} (\sigma_z=1mm)$	[V/pC/cm <sup>2</sup> ]	0.22	0.53	0.23	0.02
$k_{\parallel} (\sigma_z=1mm)$	[V/pC]	1.36	1.71	1.46	0.85

### 3. Criteria for Cavity Design: accelerating cells

## Evolution of inner cells proposed for ILC

TESLA optimized

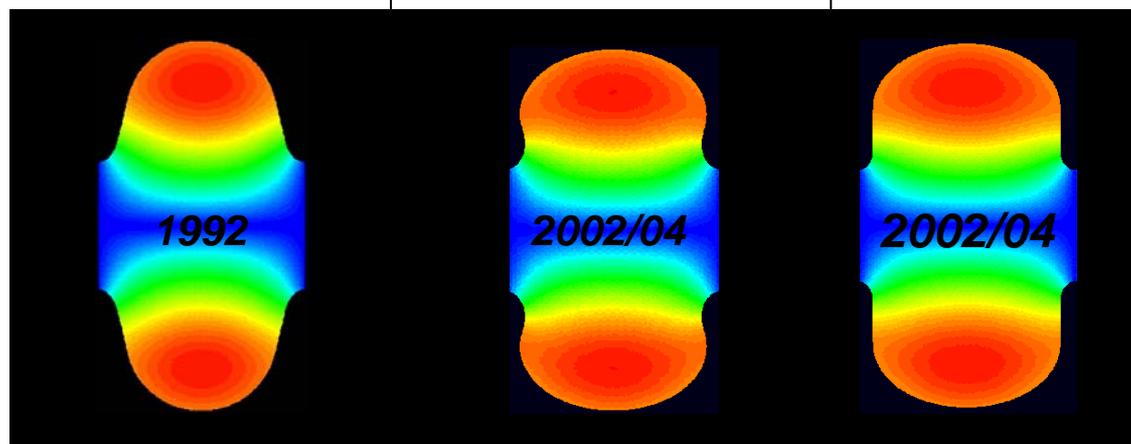
$$E_{peak}/E_{acc}$$

Re-entrant optimized

$$B_{peak}/E_{acc}$$

LL optimized

$$B_{peak}/E_{acc}$$



$r_i$	[mm]	35	30	30
$k_{cc}$	[%]	1.9	1.56	1.52
$E_{peak}/E_{acc}$	-	1.98	2.30	2.36
$B_{peak}/E_{acc}$	[mT/(MV/m)]	4.15	3.57	3.61
$R/Q$	[ $\Omega$ ]	113.8	135	133.7
$G$	[ $\Omega$ ]	271	284.3	284
$R/Q * G$	[ $\Omega * \Omega$ ]	30840	38380	37970
$k_{\perp}$ ( $\sigma_z=1mm$ )	[V/pC/cm <sup>2</sup> ]	0.23	0.38	0.38
$k_{\parallel}$ ( $\sigma_z=1mm$ )	[V/pC]	1.46	1.75	1.72

### 3. Criteria for Cavity Design: **deflecting cells**

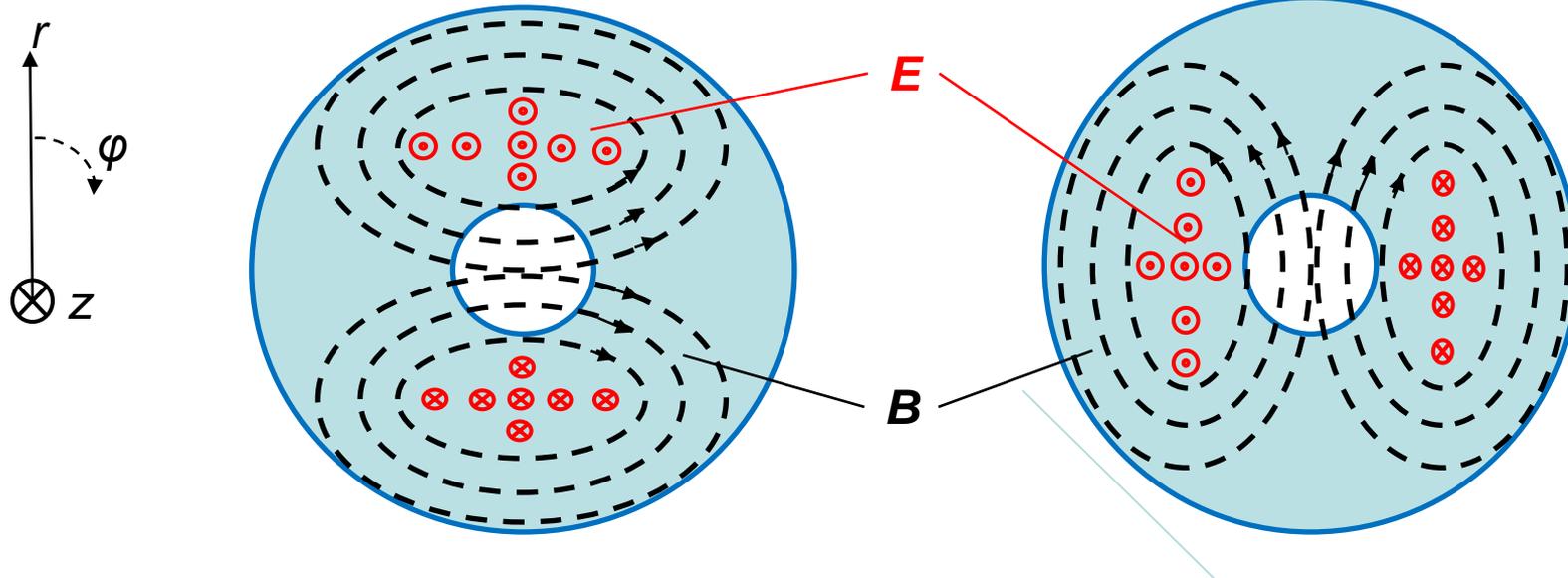
As already mentioned deflecting (crab) TM cavities operate in  $TM_{110}$ -like dipole mode.

For cylindrical (or close to cylindrical) symmetry, all non-monopole modes; dipoles quadrupoles ... are split in two polarizations (two modes shifted by  $90^\circ$ ).

#### TM<sub>110</sub> dipole

Polarization 1

Polarization 2

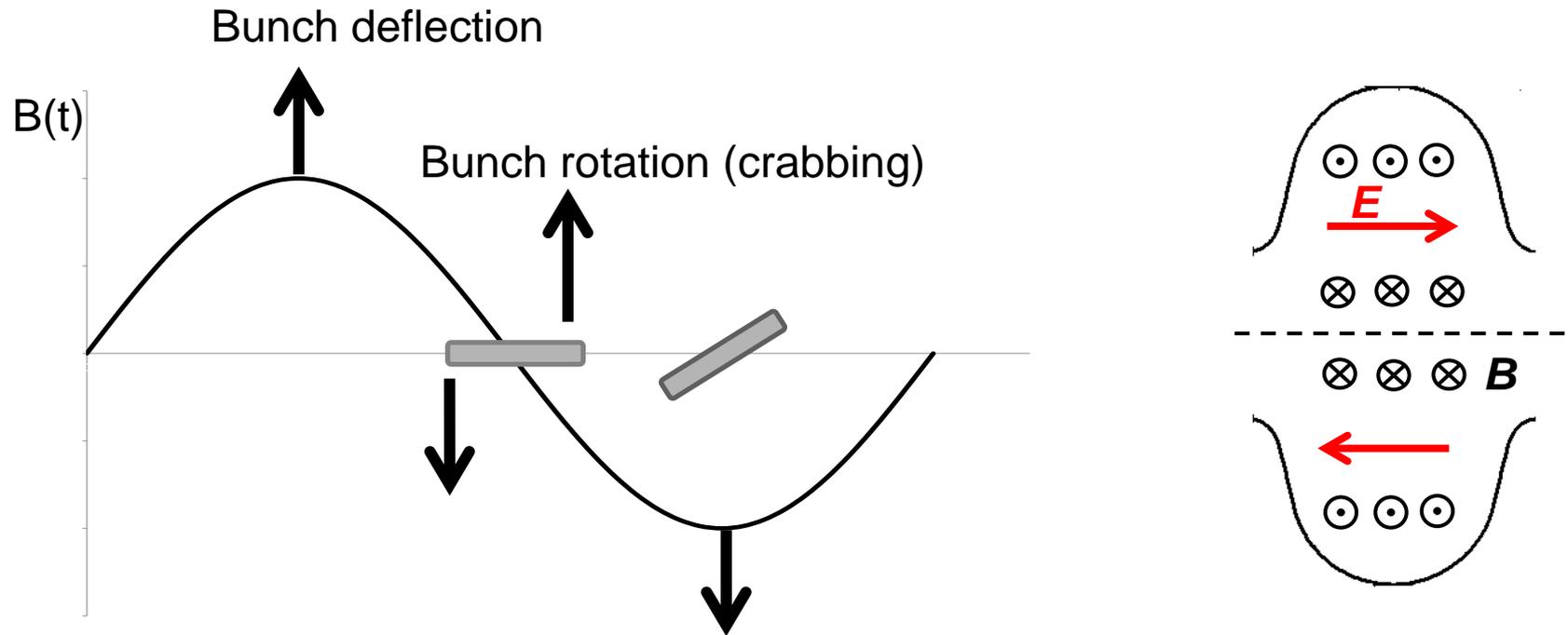


For a cavity having **perfect** cylindrical symmetry polarizations have the same frequency.

For a cavity having **perturbed** cylindrical symmetry the frequencies differ, more for more deformed shapes.

### 3. Criteria for Cavity Design: deflecting cells

Bunch rotation (crabbing) and/or deflection

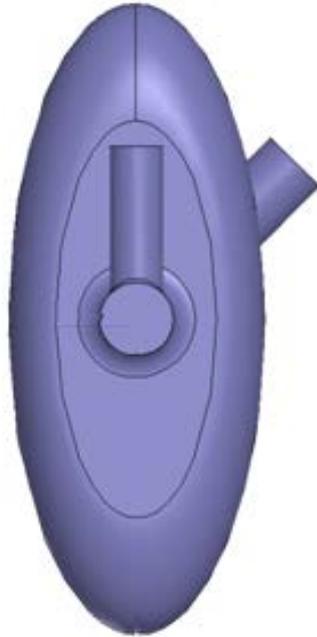


For a crab cavity additional criteria need to be taken into account:

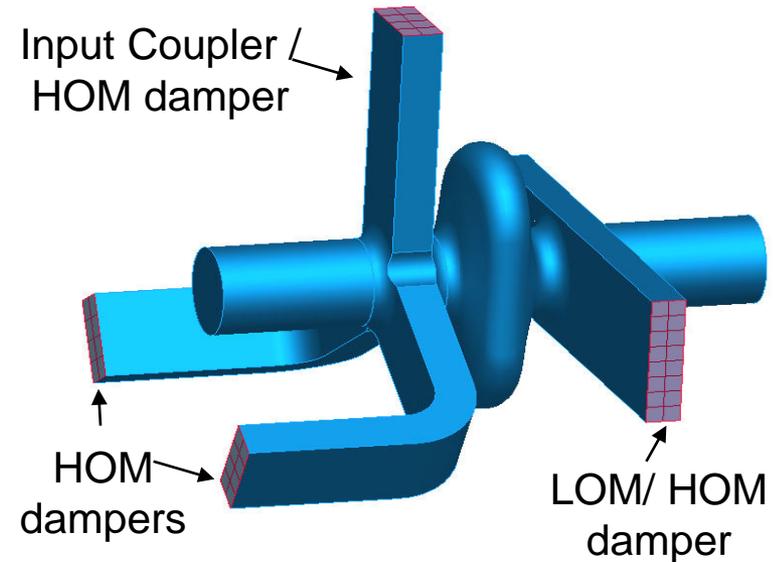
1. Angular position of the deflecting mode must be fixed.
2. Frequency of the second polarization must be apart from the deflecting frequency.
3. Lower-, Same- and Higher order modes must be well damped.

### 3. Criteria for Cavity Design: deflecting cells

Both fixing position of the deflecting mode and frequency difference is often achieved by per purpose deformation of cells.



800 MHz CC for LHC  
L. Ficcadenti, J. Tuckmantel et al.



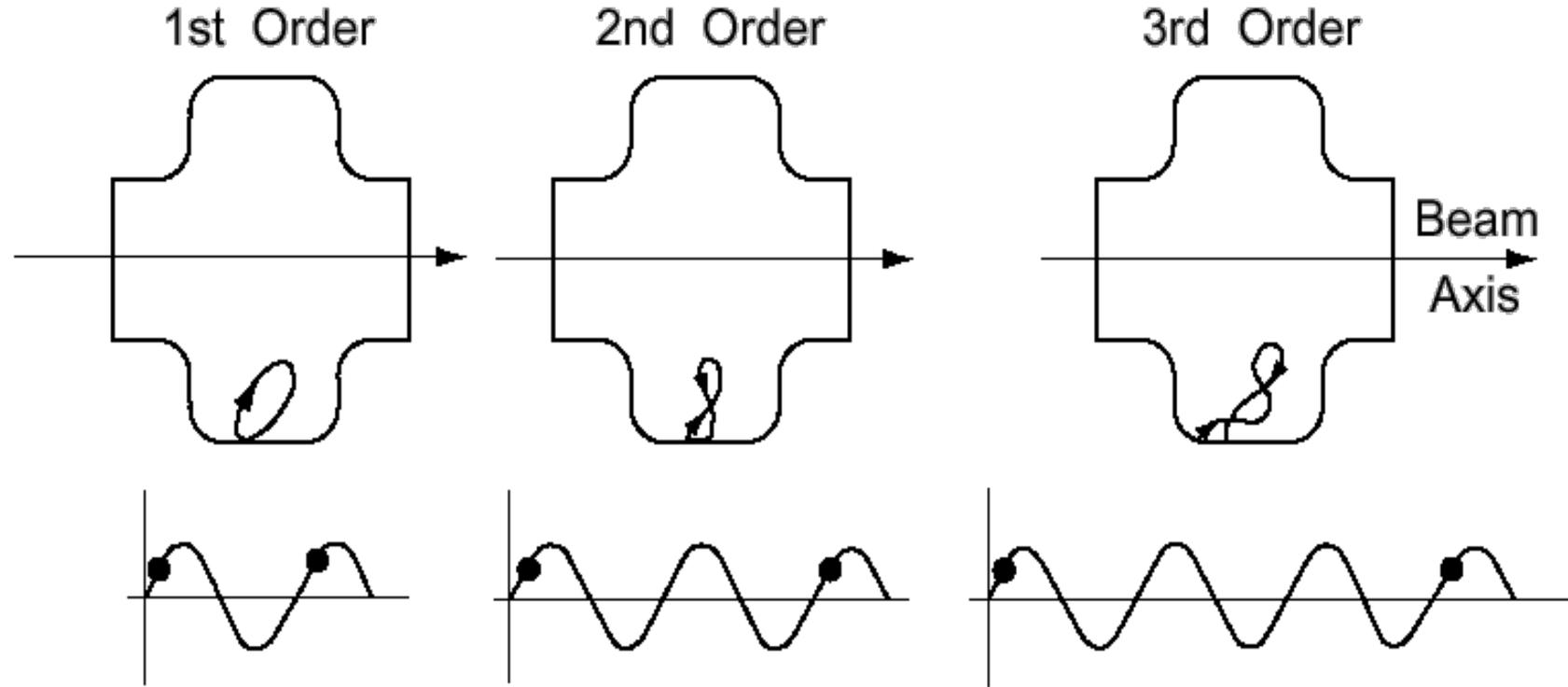
2.8 GHz CC for APS  
H. Wang/A. Nassiri et al. ANL/JLab/LBL

As we can see, the LOM, SOM and HOM damping is rather demanding for C-cavities

# 3. Criteria for Cavity Design

## Multipacting

It is a phenomenon of resonant electron emission and multiplication.

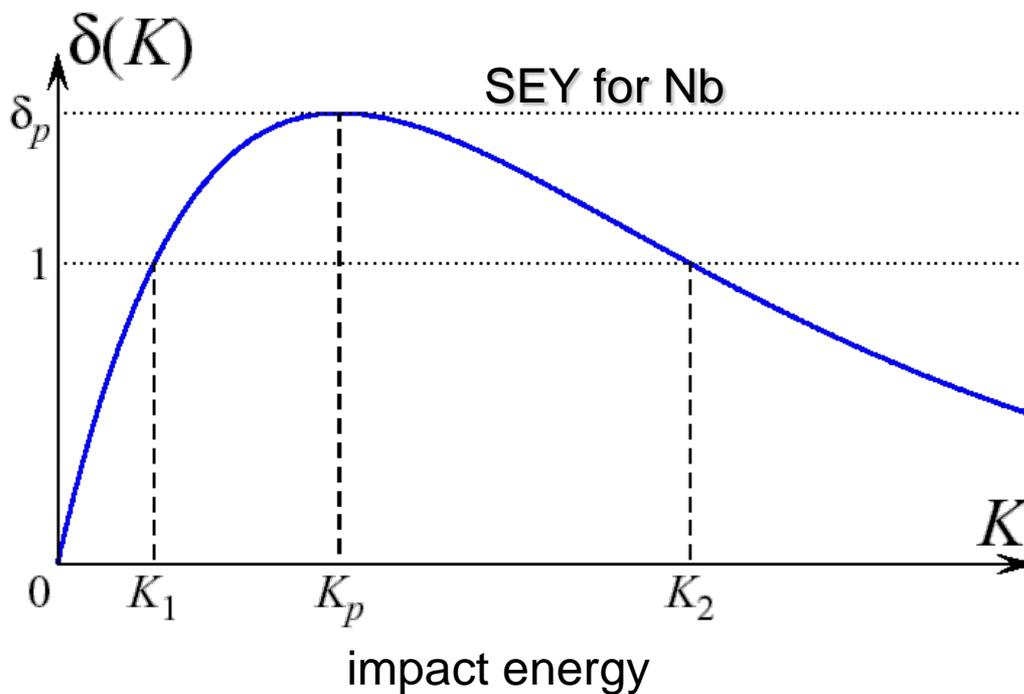


Curtsey W. Hartung

Impacting electron might create more than one secondary electron. This depends on the impact energy  $K$  and secondary emission yield  $\delta(K)$ .

### 3. Criteria for Cavity Design

SEY is function of the impact energy  $K$  and depends on the surface cleanliness.



Condition	$K_1$	$K_2$
high SEY	$\sim 27$ eV	$\gtrsim 2000$ eV
typical SEY	$\sim 40$ eV	$\sim 1000$ eV
low SEY	$\sim 150$ eV	$\sim 750$ eV

When happens, multipacting is barrier in rising the accelerating field in cavities and usually leads to quench.

In the design process we need to prove whether or not the shape of cell allows for multipacting.

## 4. Multi-cell Structures and Weakly Coupled Structures

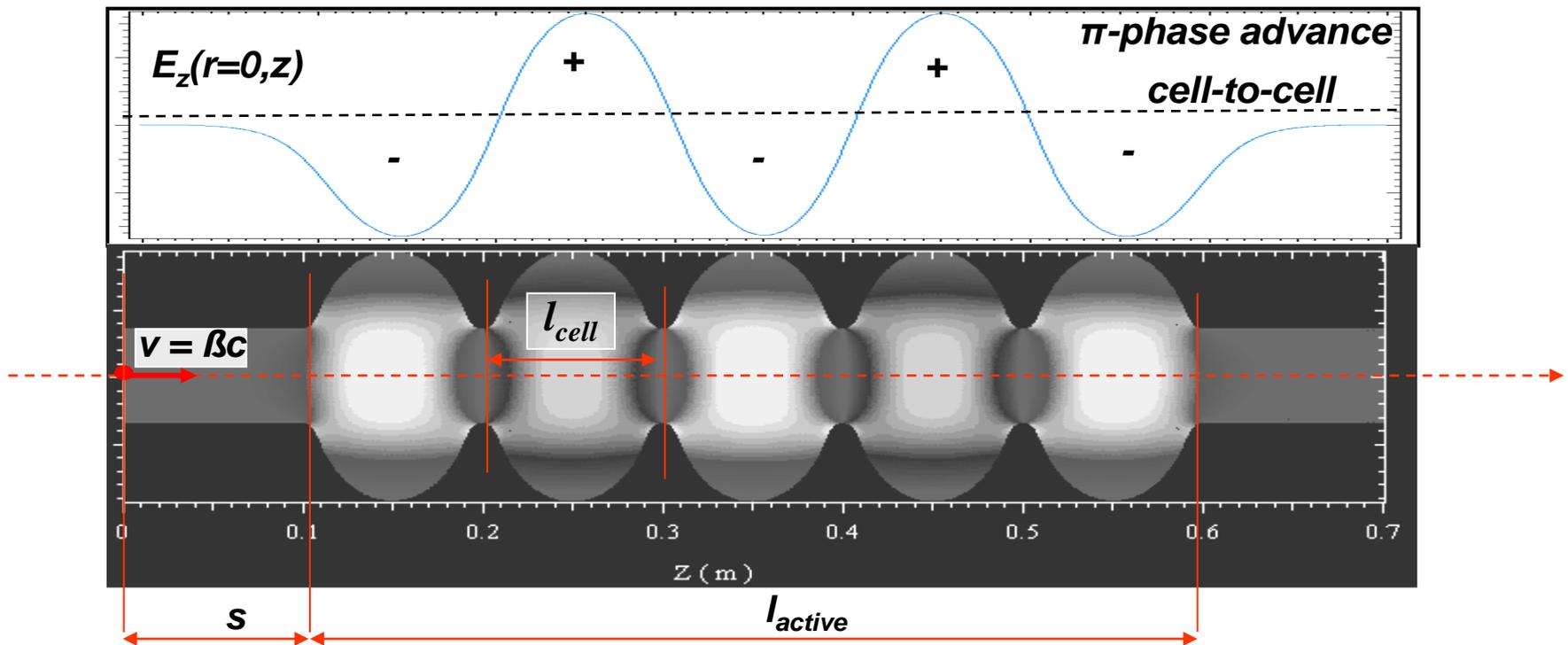
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We re-call **pros** and **cons** for a multi-cell structure:

- ◆ Cost of accelerators is lower (less auxiliaries: LHe vessels, tuners, fundamental power couplers, control electronics.....)
- ◆ Higher real-estate gradient (better fill factor )
- ◆ Field flatness vs.  $N$
- ◆ HOM excitation and trapping vs.  $N$
- ◆ Power capability of fundamental power couplers vs.  $N$
- ◆ Surface cleaning procedures become more complicated
- ◆ The worst performing cell limits whole multi-cell structure

## 4. Multi-cell Structures and Weakly Coupled Structures

### Accelerating mode in multi-cell structures



Synchronic acceleration of relativistic particles in a multi-cell structure takes place when:

1.  $l_{active} = Nl_{cell} = Nc\beta/(2f)$  and
2. the injection takes place at an optimum phase  $\varphi_{opt}$ , which ensures that particles arrive at the mid-plane of the first cell when  $E_{acc}$  reaches its maximum.

## 4. Multi-cell Structures and Weakly Coupled Structures

### Field flatness in multi-cell structures

	<i>Original Cornell</i> $N = 5$	<i>High Gradient</i> $N = 7$	<i>Low Loss</i> $N = 7$	<i>TESLA</i> $N = 9$	<i>SNS</i> $\beta = 0.61$ $N = 6$	<i>SNS</i> $\beta = 0.81$ $N = 6$	<i>Low Loss</i> $N = 9$	<i>RHIC</i> $N = 5$
<i>year</i>	1982	2001	2002	1992	2000	2000	2003	2003
$a_{ff}$	1489	2592	3288	4091	3883	2924	5435	850


$$a_{ff} = \frac{N^2}{k_{cc} \cdot \beta}$$

Decades of experience with: heat and chemical treatment, handling and assembly allow to maintain good field profile, even in cavities with bigger  $N$  and weaker  $k_{cc}$ .

For many TESLA cavities: field flatness is better than 95 %

## 4. Multi-cell Structures and Weakly Coupled Structures

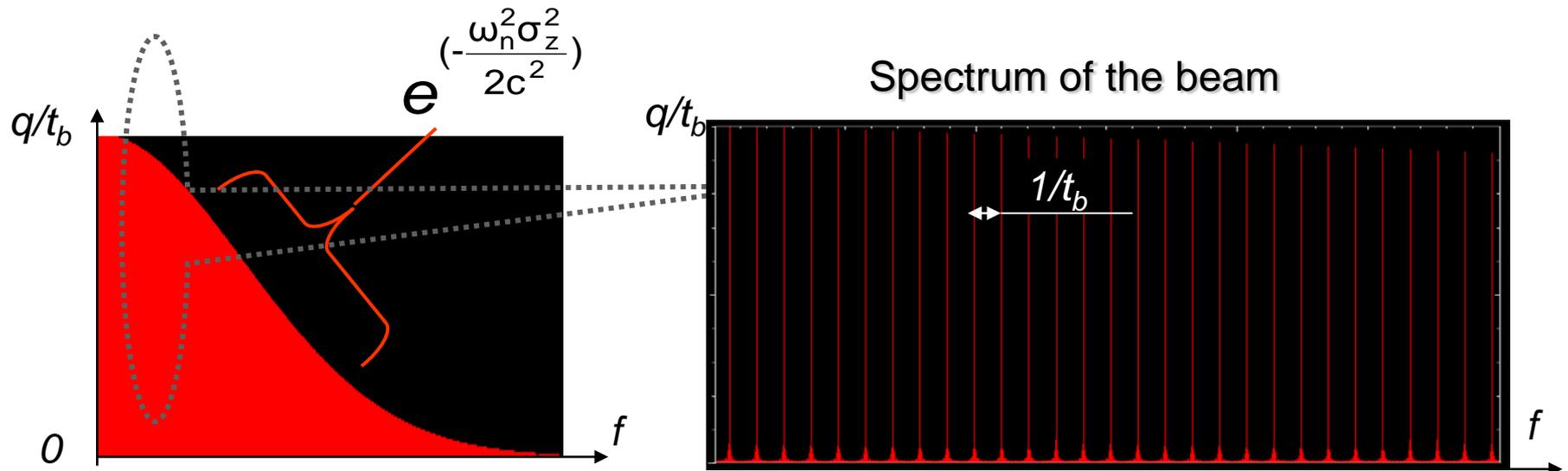
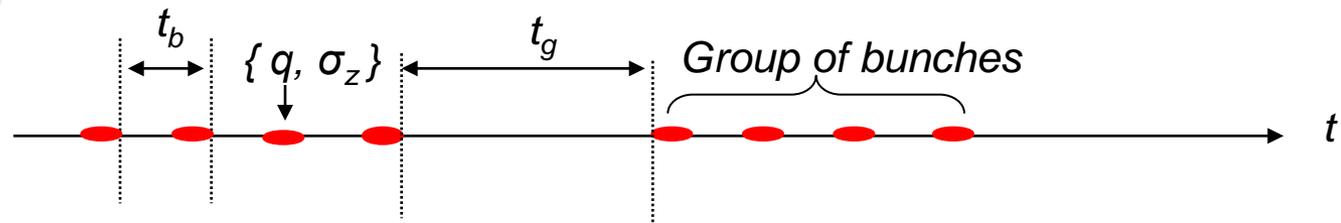
### HOM excitation and HOM trapping in multi-cell structures

HOM excitation causes:

- Beam instabilities and/or dilution of emittance
- Bunch-to-bunch energy modulation
- Additional cryogenic loss

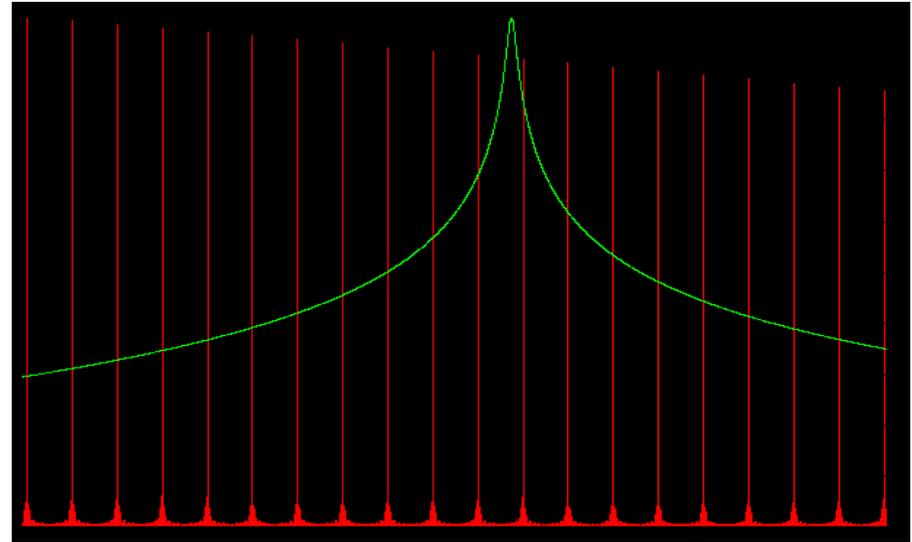
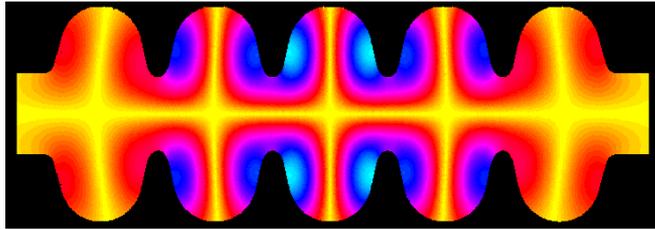
HOM excitation:

Time structure of the beam



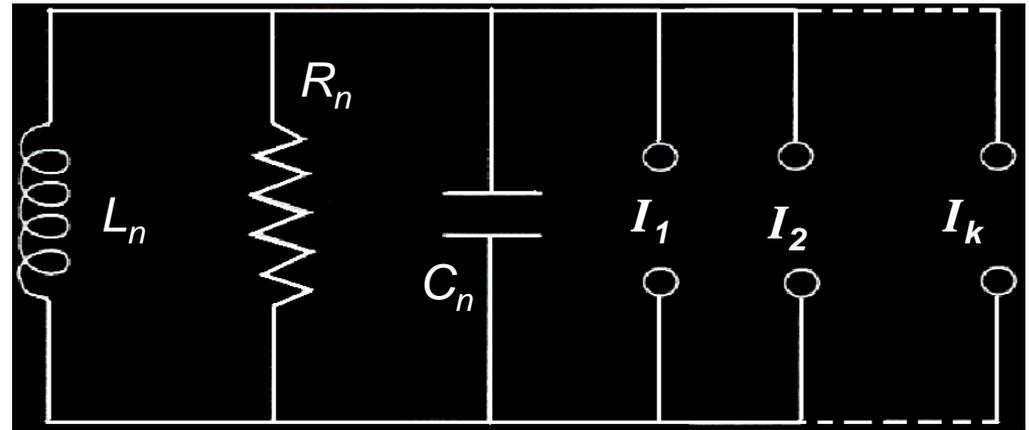
# 4. Multi-cell Structures and Weakly Coupled Structures

Mode No.  $n$  :  $\{ \omega_n, (R/Q)_n, Q_{L,n} \}$



$$Z_n(\omega) = \frac{(R/Q)_n \cdot Q_{L,n}}{1 + jQ_{L,n} \left( \frac{\omega}{\omega_n} - \frac{\omega_n}{\omega} \right)}$$

Impedance of the  $n$ -mode



Multi-source excitation

## 4. Multi-cell Structures and Weakly Coupled Structures

The power induced by “all” spectral lines (current sources) in mode No.  $n$ :

$$P_n = \frac{1}{2} \sum_k Z_n(\omega_k) \cdot I_k^2$$
$$Z_n(\omega) = \frac{(R/Q)_n \cdot Q_{L,n}}{1 + jQ_{L,n} \left( \frac{\omega}{\omega_n} - \frac{\omega_n}{\omega} \right)}$$

where:  $\frac{1}{Q_{L,n}} = \frac{1}{Q_{0,n}} + \frac{1}{Q_{ext,n}}$  ← Measure of the extracted power

The HOM couplers, devices extracting the energy from parasitic modes, are attached to cavities for mitigation of high and harmful  $E$ - $M$  fields of HOM .

The experience is that, the HOM couplers can be attached to the beam tubes and must not be located at cells because this leads to the performance degradation.



## 4. Multi-cell Structures and Weakly Coupled Structures

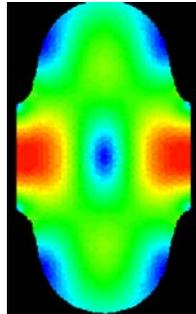


Waveguide  
HOM ports

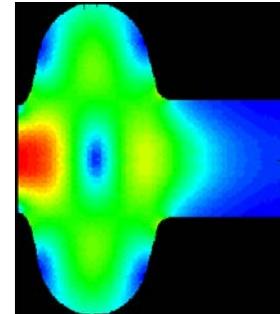
The HOM trapping is similar to the FM field profile unflatness mechanism:

- weak  $k_{cc,HOM}$ , cell-to-cell coupling for HOM
- difference in the HOM frequency between the end-cells and inner-cells

$f = 2385 \text{ MHz}$



That is why they  
hardly resonate  
together

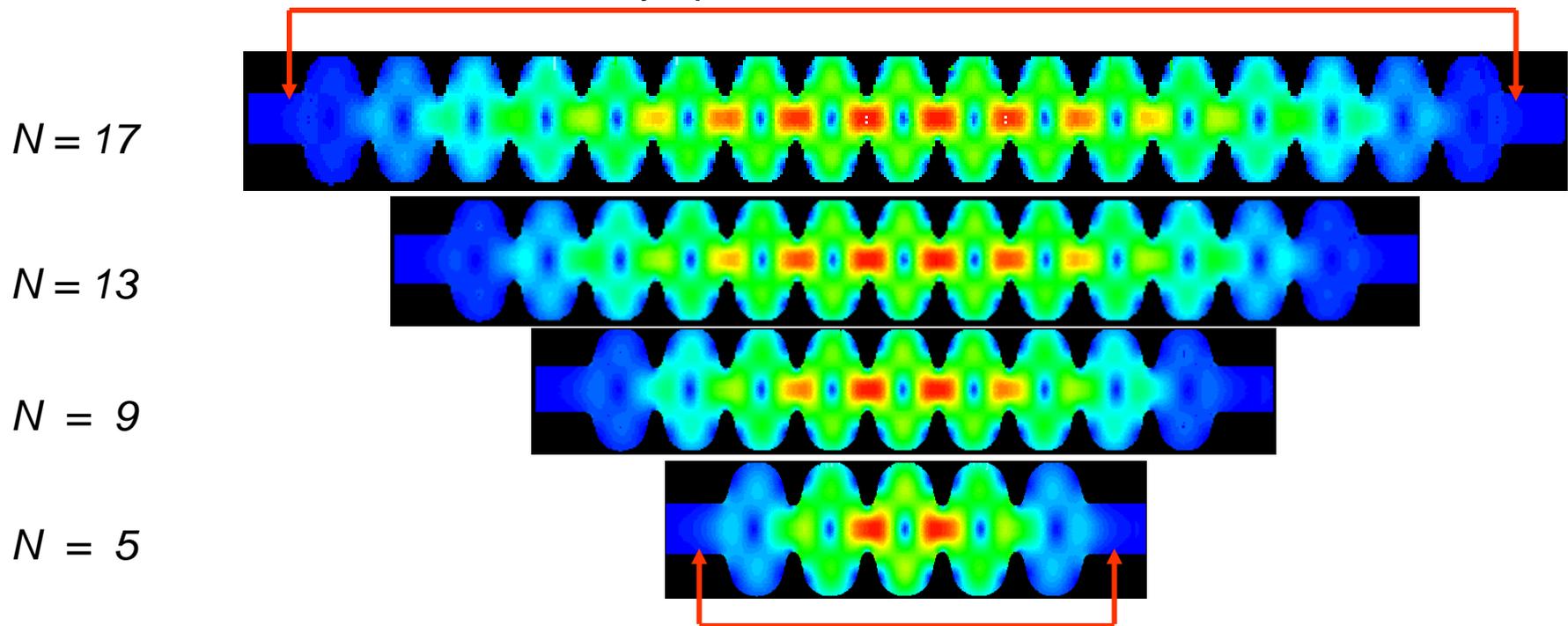


$f = 2415 \text{ MHz}$

## 4. Multi-cell Structures and Weakly Coupled Structures

Example of the trapping and how  $N$  influences strength of the  $E-H$  fields at the HOM couplers locations

no  $E-H$  fields at HOM couplers locations (trapping), which are always placed at the end beam tubes



$E-H$  fields at HOM couplers locations

Less cells in a structure helps always to reach lower Qs of HOMs.

## 4. Multi-cell Structures and Weakly Coupled Structures

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What else can help to avoid the trapping?

### Adjustment of end-cells

The geometry of end-cells differs from the geometry of inner cells due to the attached beam tubes, HOM- and input couplers.

Their function is multifold and their geometry must fulfill three requirements:

- field flatness and frequency of the accelerating mode
- field strength of the accelerating mode must enable matching of  $Q_{ext}$  of FPC
- field strength of the dangerous HOMs must ensure required damping.

All three requirements make design of the end-cells more difficult than inner cells.

## 4. Multi-cell Structures and Weakly Coupled Structures

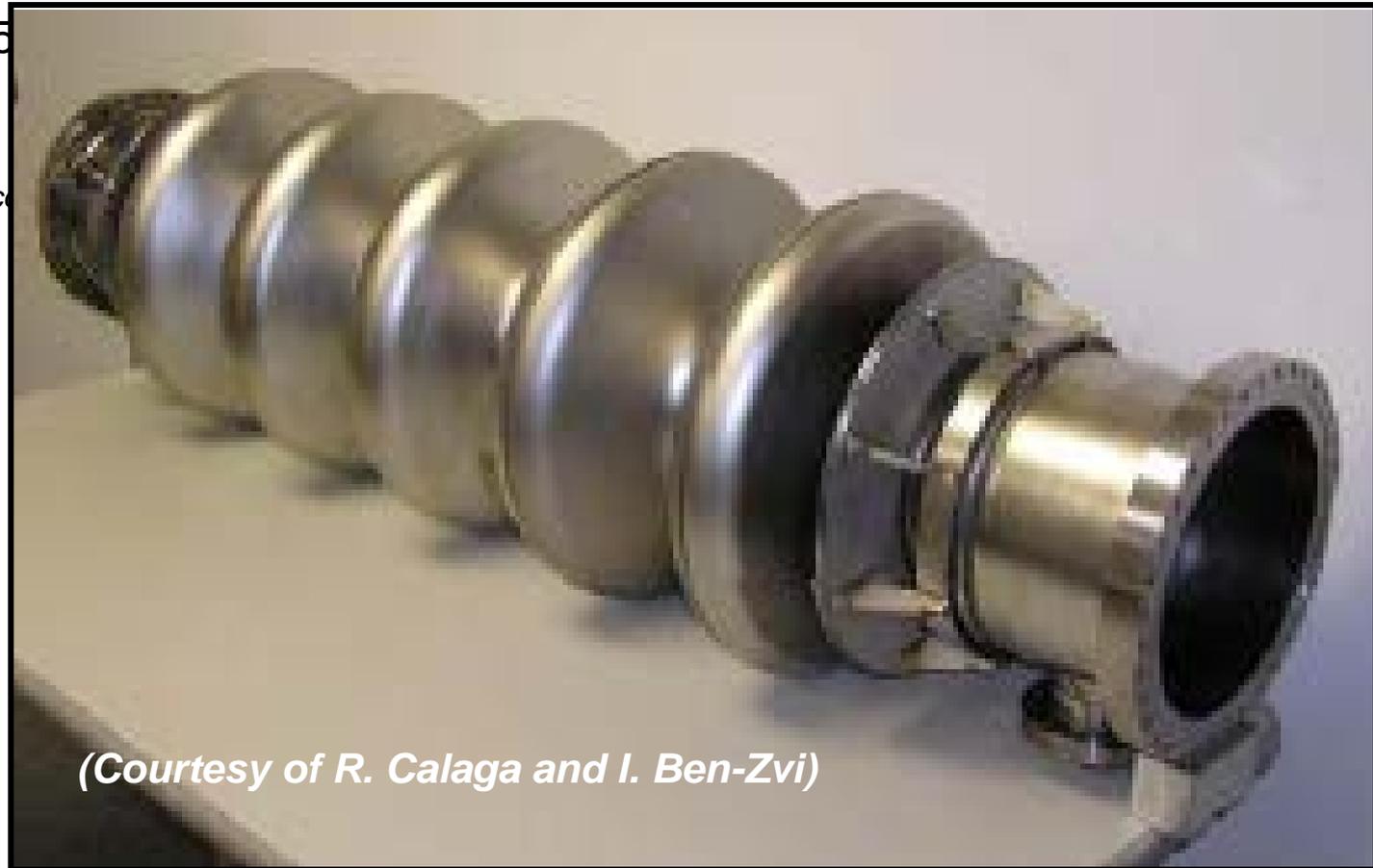
1. Open irises of the inner- and end-cells (bigger  $k_{cc,HOM}$ ) and shaping them similarly.

Example: RHIC 5

Monopole mode  $k_c$

$f_{HOM} = 1394$  MHz

$f_{HOM} = 1403$  MHz



The method causes (R/Q) reduction of fundamental mode, which in this application is less relevant.

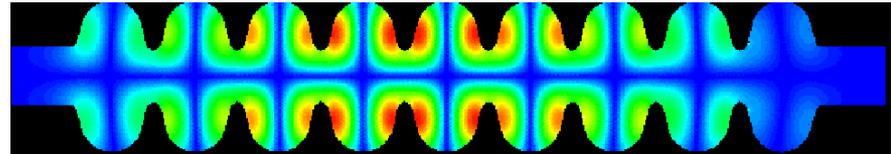
# 4. Multi-cell Structures and Weakly Coupled Structures

2. Tailoring the end-cells to equalize HOM frequencies of the inner- and end-cells.

Example: TESLA 9-cell cavity, which has two different end-cells (asymmetric cavity)

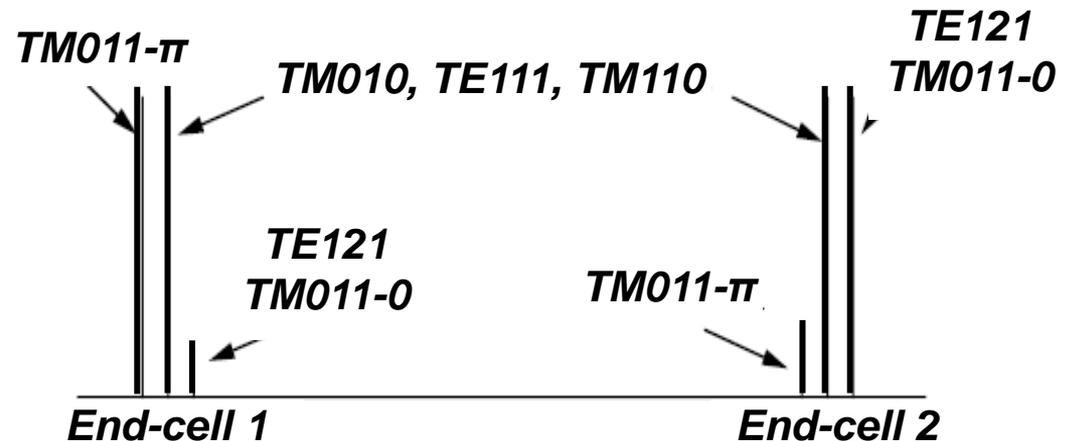
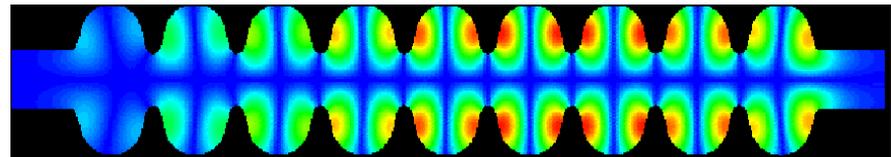
The lowest mode in the passband

$$f_{HOM} = 2382 \text{ MHz } \mathbf{TM011-\pi}$$



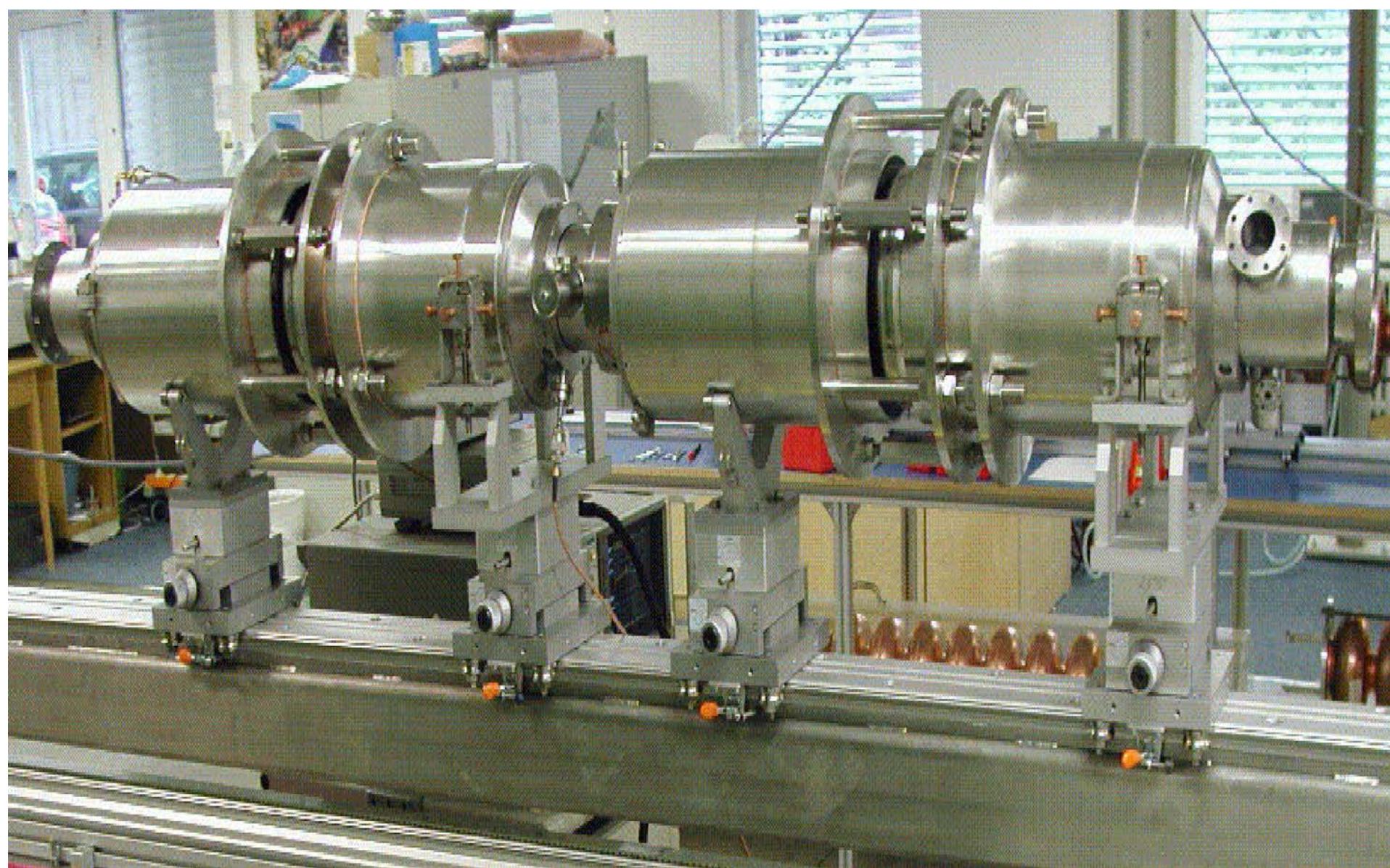
The highest mode in the passband

$$f_{HOM} = 2458 \text{ MHz } \mathbf{TM011-0}$$



The method works for few modes but keeps the (R/Q) of the fundamental mode high.

## 4. Multi-cell Structures and Weakly Coupled Structures



## 4. Multi-cell Structures and Weakly Coupled Structures

### Power capability of the FPC for multi-cell structures

When  $I_{beam}$  and  $E_{acc}$  are specified and for a superconducting multi-cell structure :

$$P_{in} \sim N$$

$Q_{ext}$  of the FPC, which usually is  $\ll$  than intrinsic  $Q_0$ , is:

$$Q_{ext} \approx \frac{E_{acc} \cdot \beta \cdot \lambda \cdot N}{I_{beam} \cdot (R/Q)_{cell} \cdot N} = \frac{E_{acc} \cdot \beta \cdot \lambda}{I_{beam} \cdot (R/Q)_{cell}} = \frac{\omega_{acc} \cdot W_{onecell} \cdot N}{\frac{1}{2} \int_{S_{inputport}} E_{acc} \times H_{acc} ds}$$

Independent of N

It must be  $\sim N$  to keep the ratio constant

# 4. Multi-cell Structures and Weakly Coupled Structures

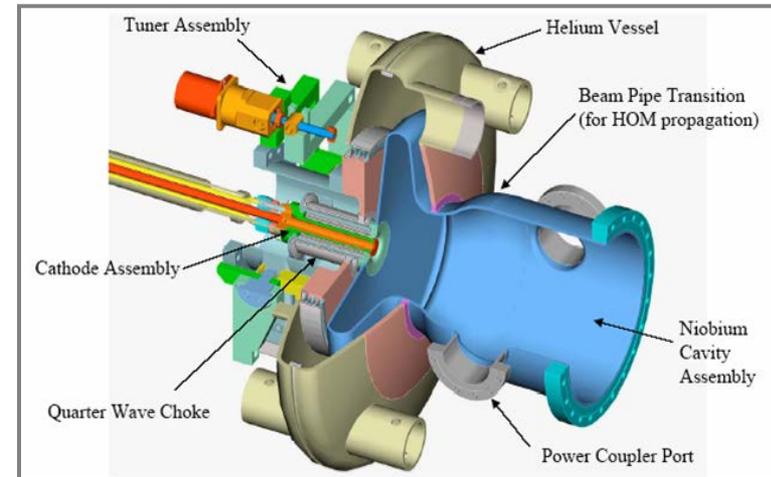
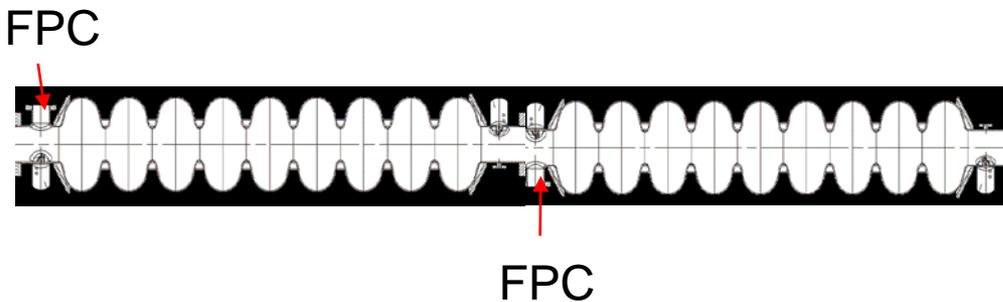
$$\int_{S_{inputport}} \mathbf{E}_{acc} \times \mathbf{H}_{acc} ds$$

Coupler must penetrate deeper in the beam tube or/and is placed closer to the end cell

Opening for the coupler must be bigger

Each method causes perturbation of the symmetry and increases kick to the beam.

The remedies are: alternating positions of couplers or double couplers



Courtesy of Alan Todd (AES)

## 4. Multi-cell Structures and Weakly Coupled Structures

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### Surface cleaning procedures are more complicated

Few words on Nb and on the surface preparation procedures

All high gradient cavities are made of the pure metallic bulk Nb (II-type sc,  $T_c = 9.2$  K):

- We use poly-crystal Nb from the very beginning.
- Recently, as proposed by P. Kneisel (JLab) we use also large-grain Nb.

Surface preparation has several steps with three major procedures:

- Chemical treatment: can be Buffered Chemical Polishing or Electro-Polishing
- Heat treatment
- High Pressure Water rinsing

#### Buffered Chemical Polishing (BCP)

Acids: HF (49%), HNO<sub>3</sub> (65%), H<sub>3</sub>PO<sub>4</sub> (85%)

Mixture: 1:1:1 or 1:1:2 by volume

#### Electro-Polishing (EP)

Electrolyte:

1 part HF(49%), 9 parts H<sub>2</sub>SO<sub>4</sub> (96%)

Al-cathode, Nb-anode,  $J \sim 50$  mA/cm<sup>2</sup>

## 4. Multi-cell Structures and Weakly Coupled Structures

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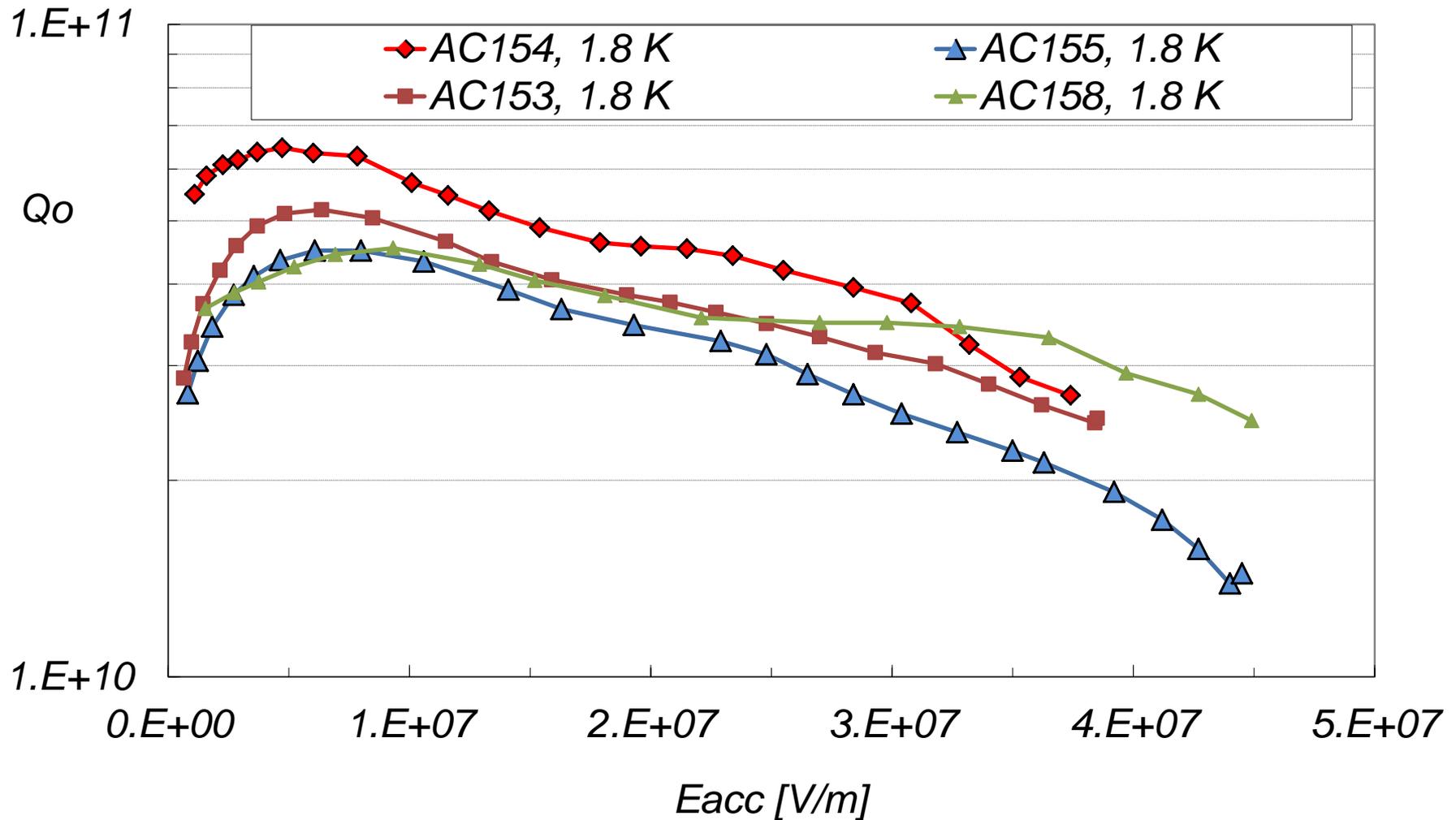
The sequence in the surface preparation is:

- Heavy chemical etch (EP or BCP)
  - Removal of damaged surface layer (100-150um) caused by fabrication and handling
- Removal of surface contamination
  - Ultrasonic cleaning of surface with detergent and DI water, or Alcohol rinse
- Heat treatment (600-800C in vacuum furnace)
  - Removes hydrogen from the bulk niobium to reduce the risk of Q-disease
- RF tuning and mechanical inspection
  - Field profile, calibration of test probes, check mechanical structure
- Removal of surface contamination
  - Ultrasonic cleaning of surface with detergent and DI water,
- Light chemical etch (EP or BCP)
  - Remove any risk from damage during handling and furnace contamination
- High pressure rinse (UPW @100 Bar) + Class 10 drying of cavity
  - Reduction of field emission sources, surface particulates
  - At least two passes over entire surface

*Courtesy J. Mammosser*

## 4. Multi-cell Structures and Weakly Coupled Structures

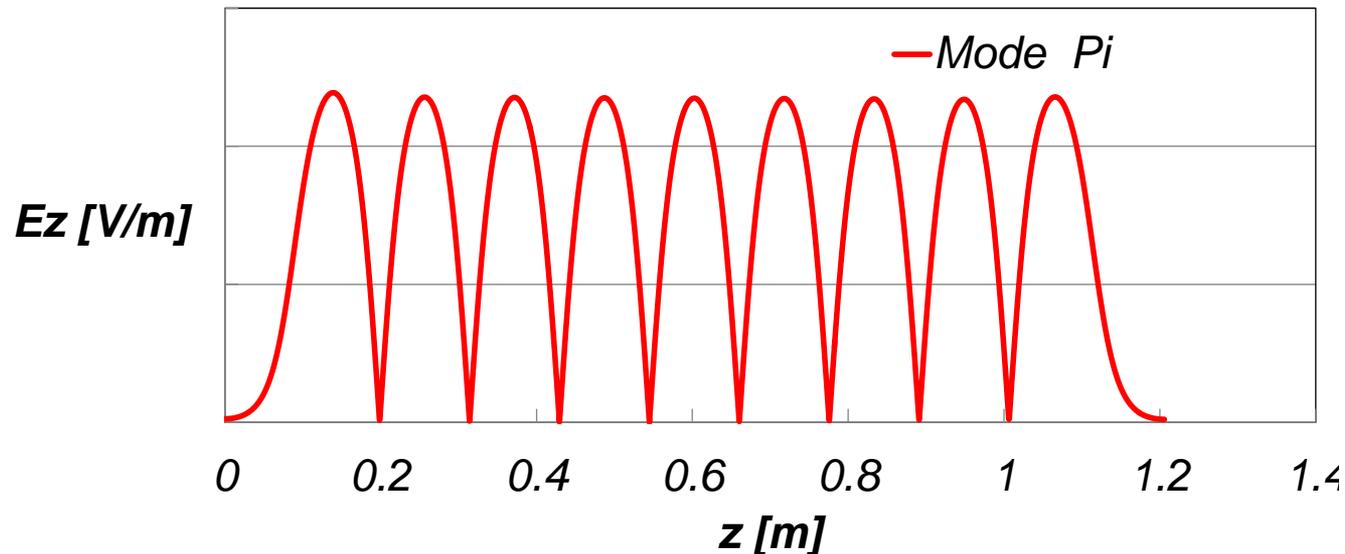
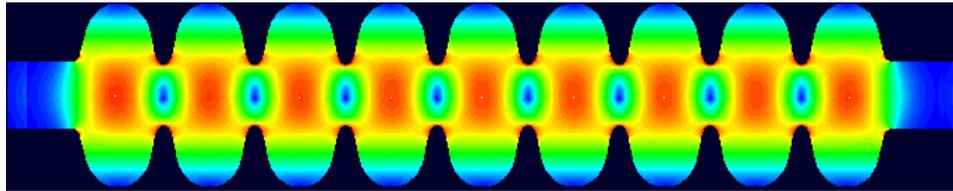
But, when you use Large Grain Nb, make proper cleaning and operate cavities at 1.8K the result can be very nice:



## 4. Multi-cell Structures and Weakly Coupled Structures

### The worst performing cell limits whole multi-cell structure

After the pre-tuning all cells have the same amplitude



We can find out which cell has the worst performance with temperature mapping.

Testing all modes in the FM passband we can find out which pair of cells has the worst performance. For the TESLA cavity the pairs are 1&9, 2&8, 3&7, 4&6.

Performance of the cell No 5 can be defined directly.

## 4. Multi-cell Structures and Weakly Coupled Structures

### List of multi-cell $\beta=1$ cavities optimized for various criteria

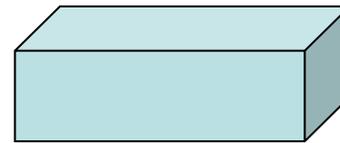
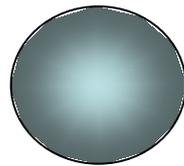
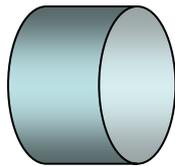
<b>Criterion</b>	<b>Structure</b>	<b>Best parameter</b>	<b>Weakest parameter (point)</b>	<b>Comments</b>
$E_{acc}$	<p>HG: 1.5 GHz, N=7</p> <p>TESLA: 1.3 GHz, N=9</p> <p>ILC-LL: 1.3 GHz, N=9</p> <p>ILC-RE 1.3 GHz, N=9</p>	<p><math>E_{peak}/E_{acc}= 1.96</math></p> <p><math>E_{peak}/E_{acc}= 1.98</math></p> <p><math>B_{peak}/E_{acc}= 3.61</math></p> <p><math>B_{peak}/E_{acc}= 3.57</math></p>	<p>maximum -Eacc</p> <p>maximum -Eacc</p> <p><math>E_{peak}/E_{acc}</math></p> <p><math>E_{peak}/E_{acc}</math></p>	<p>Designed for</p> <p><math>I_{beam} &lt; 10 \text{ mA}</math>,</p> <p>Pulse operation</p>
$P_{loss}$	<p>LL: 1.5 GHz,</p> <p>N= 7</p>	<p><math>B_{peak}/E_{acc}= 3.7</math></p> <p><math>(R/Q) \cdot G</math></p>	<p>Not easy to clean,</p> <p>HOM damping</p>	<p>Designed for</p> <p><math>I_{beam} &lt; 1 \text{ mA}</math></p> <p>First LL-type cavity</p>
$Z_{HOM}$	<p>RHIC: 0.7 GHz,</p> <p>N= 5</p>	<p>Very low: <math>k_{\perp}</math>, <math>k_{\parallel}</math></p> <p><math>E_{peak}/E_{acc}= 1.98</math></p>	<p>Cryogenic losses</p>	<p>First cavity for</p> <p><math>I_{beam} \approx 2 \text{ A}</math></p>

## 5. Tools for RF-design

Design of an elliptical cavity is usually performed in two steps: “2D” and “3D” :

- “2D” is fast and allows to define geometry of a cylindrical symmetric body (inner and end-cells) of the cavity.
- “3D” is much more time consuming but necessary for modeling of full equipped cavity with FPC and HOM couplers.  
Also coupling strength for FPC and damping of HOMs can be modeled only in 3D.

Solutions to 2D (or 3D) Helmholtz equation can be found analytically only for very few geometries (pillbox, spherical- and rectangular resonator)



We need numerical methods:  $(\nabla^2 + \omega^2 \epsilon \mu) \mathbf{A} = 0$

Approximating operator  
(Finite Difference Methods)

Approximating function  
(Finite Element Methods)

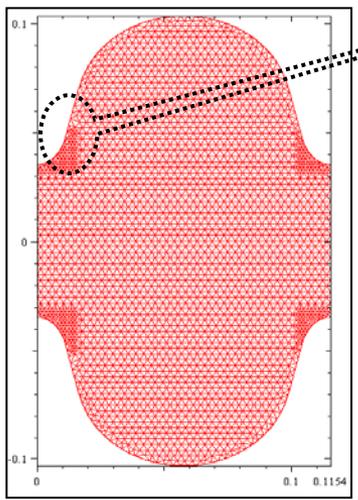
# 5. Tools for RF-design

The FEM is superior in mapping of curvilinear boundary. This is essential for modeling of:

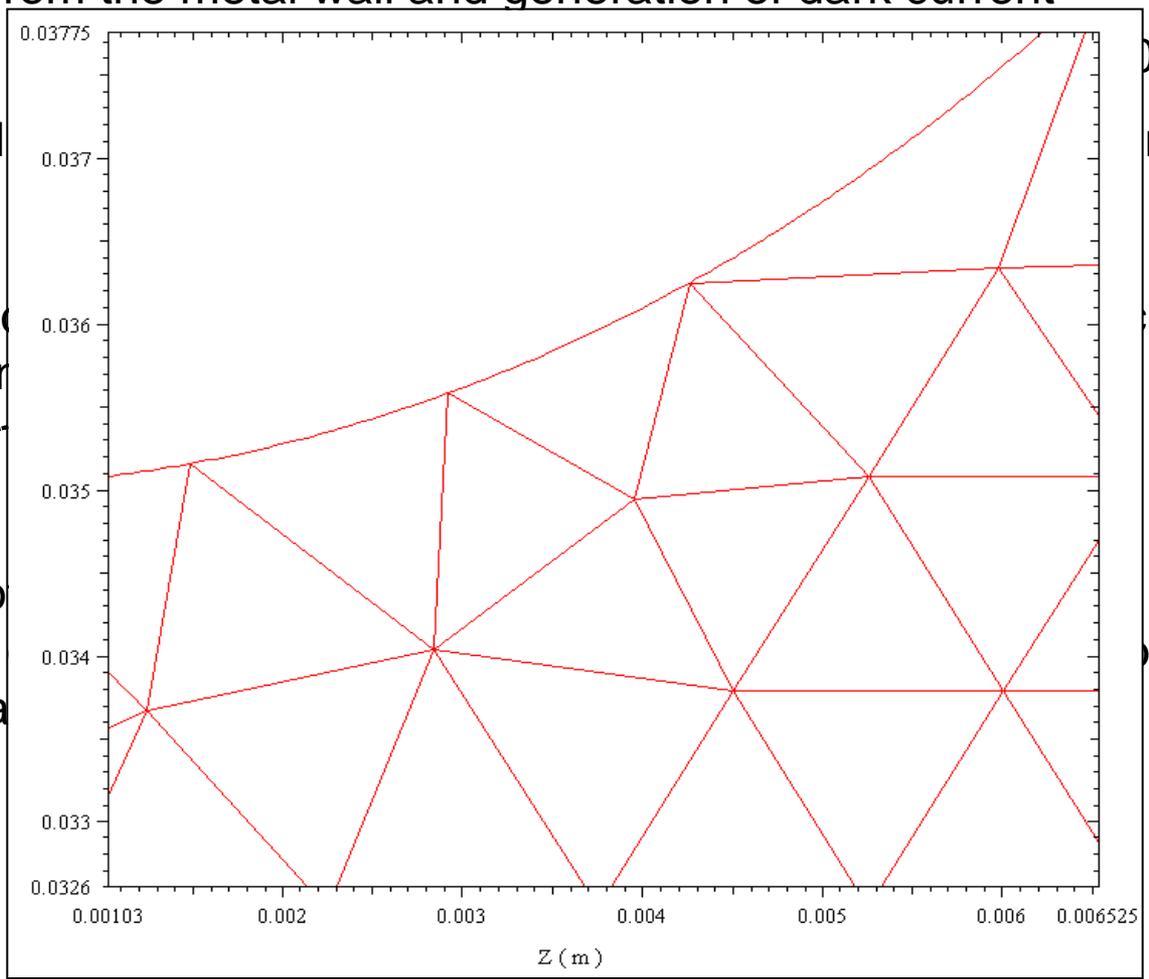
- Multipacting
- Electron emission from the metal wall and generation of dark current
- Frequency change

2D codes like SUPERFISH (2D approx.) or FEM-code (3rd

Example from the FEM code MultiPac by P. Yla-Oijala and



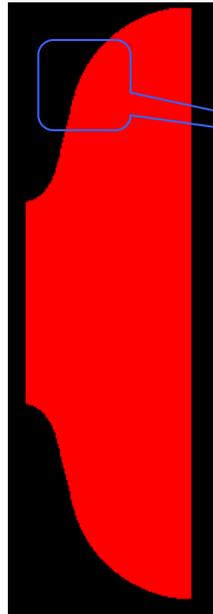
Smooth force "zigzag"



0.006525  
0.006  
0.005  
0.004  
0.003  
0.002  
0.00103  
0.03775  
0.037  
0.036  
0.035  
0.034  
0.033  
0.0326  
the  
by a

# 5. Tools for RF-design

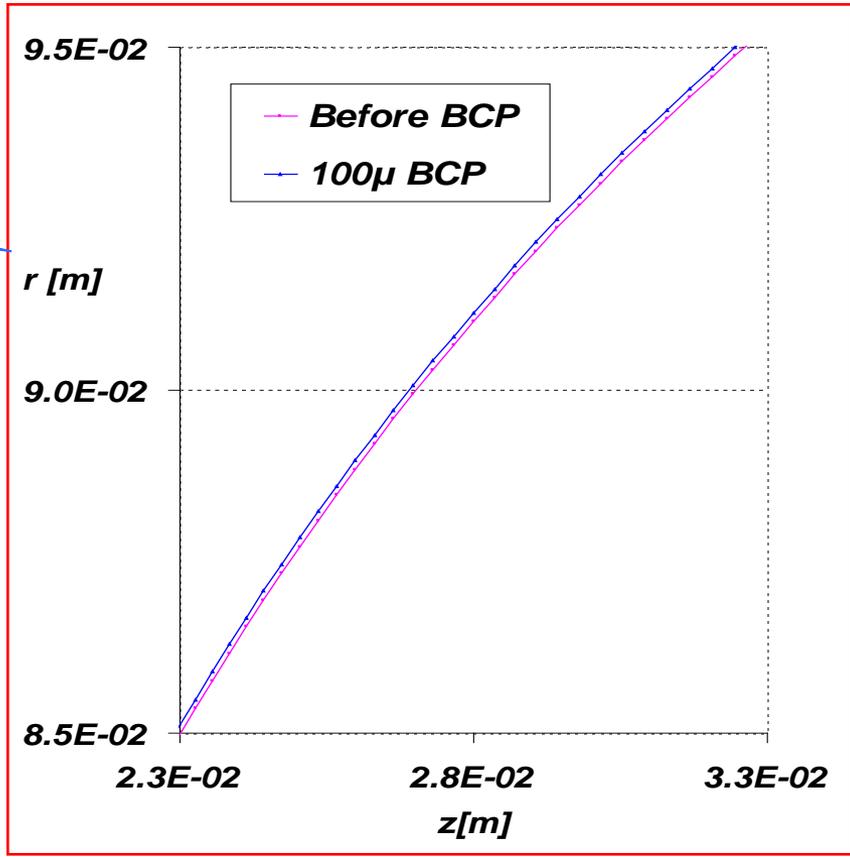
Example: FEM-code modeling of the frequency change due to the chemical treatment (removed layer of 100μm)



2D Modeling takes ~2min

$$\frac{\partial f}{100\mu m} = -10kHz / \mu m$$

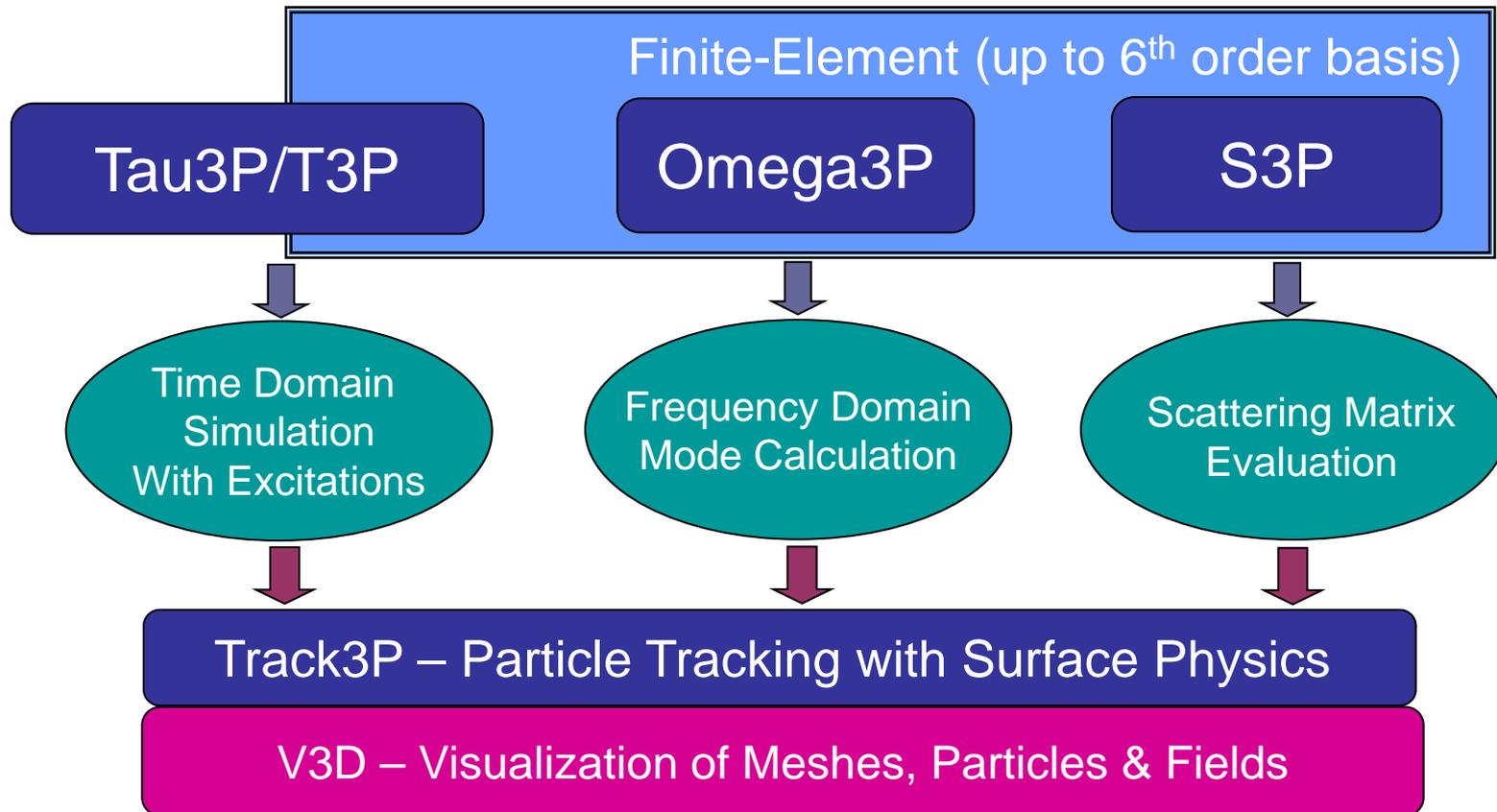
which was measured for the uniform removal



Zoomed difference in shape of the TESLA mid cup after 100μm BCP

# Electromagnetic Code Development at SLAC by ACD

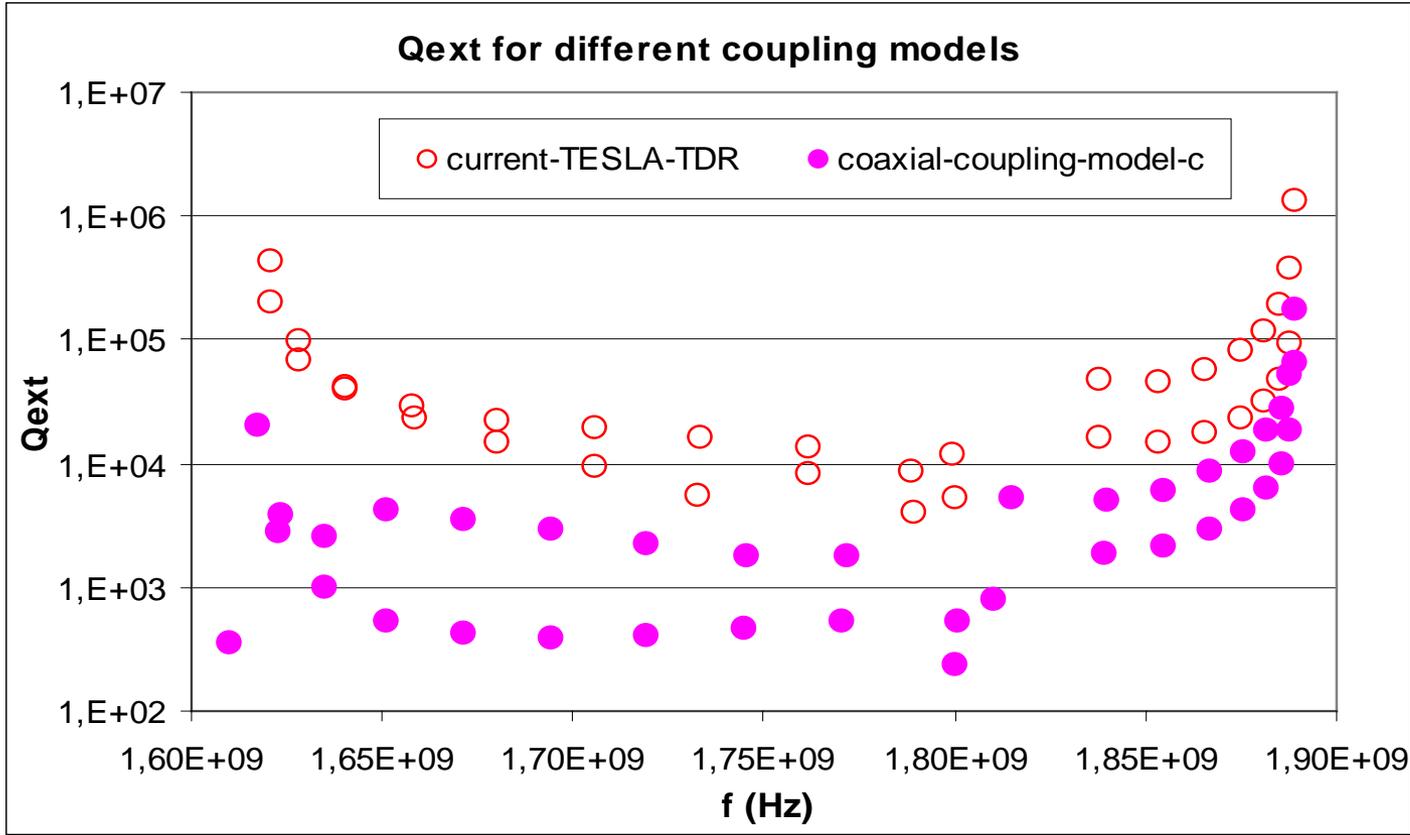
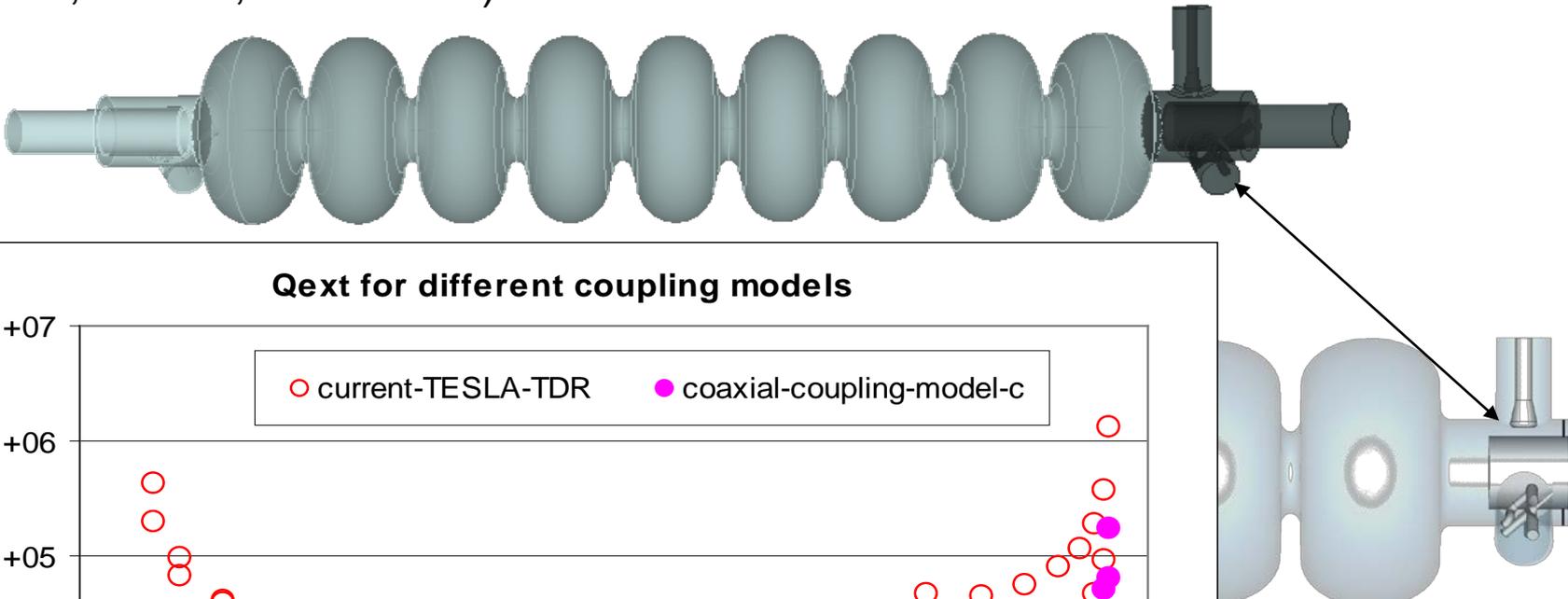
Solves Maxwell's equations with particles in TD & FD



*(Courtesy of Kwok Ko and ACD Members)*

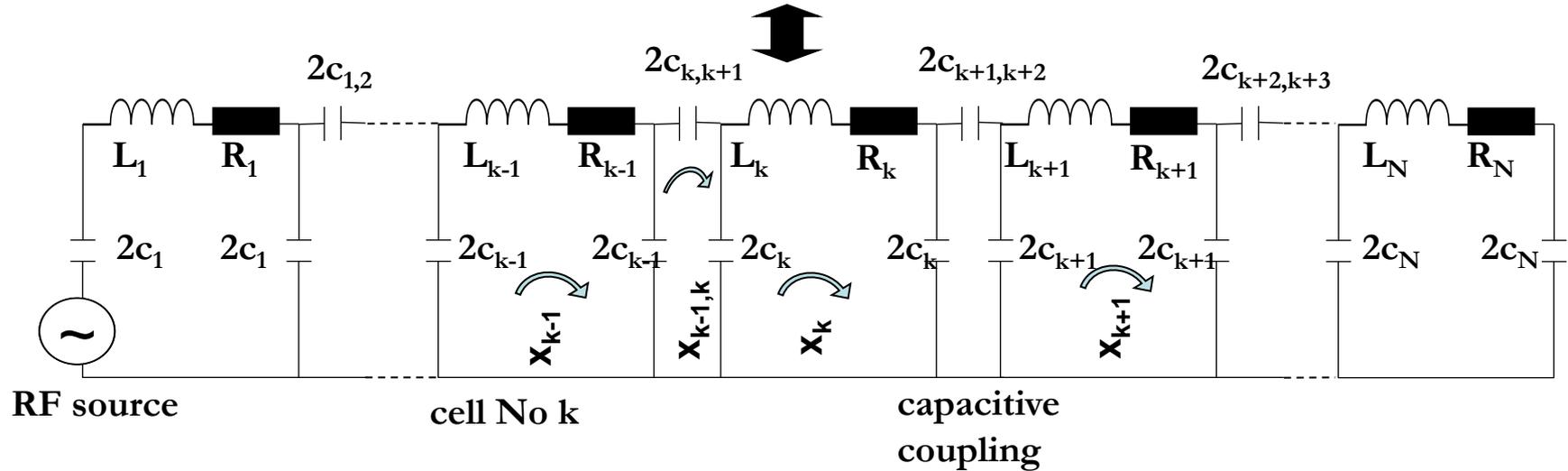
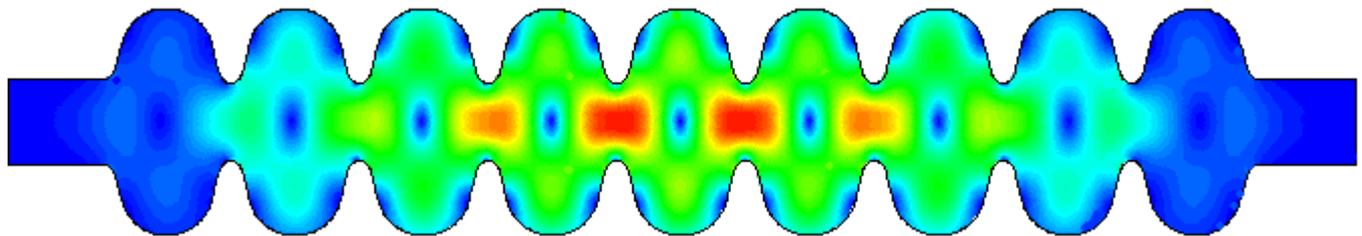
# 5. Tools for RF-design

Example of 3D dipole damping modeling for the TESLA cavity with the coaxial coupling (Omega3P, L. Xiao, ACD SLAC)



# 6. LEC and Transient State

In the design process we use 2D-codes (*SUPERFISH, SLANS, FEM..*) and 3D-codes (MWS, HFSS, MAFIA and OMEGA-3P) but still the Lumped Element replacement Circuit is helpful to investigate some RF properties.



Where:  $2\pi f_{FM} = (L_k \cdot c_k)^{-0.5}$ ;  $(R/Q)_{FM} = (L_k/c_k)^{0.5}$ ;  $R = (R/Q)_{FM} \cdot Q_{L,FM}$ ;

## 6. LEC and Transient State

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What can be done by means of the LEC:

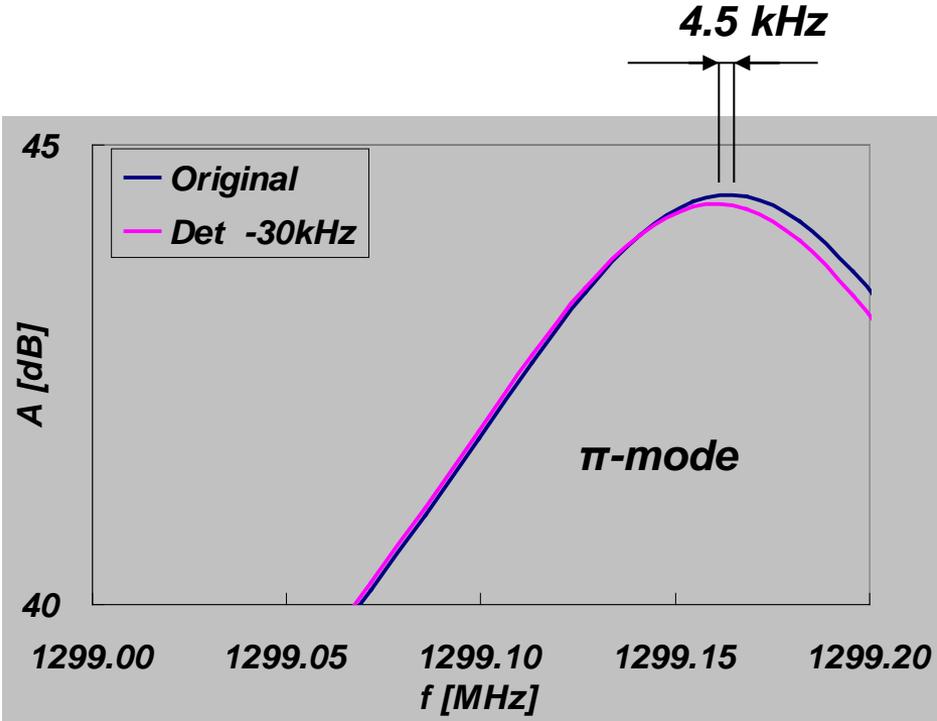
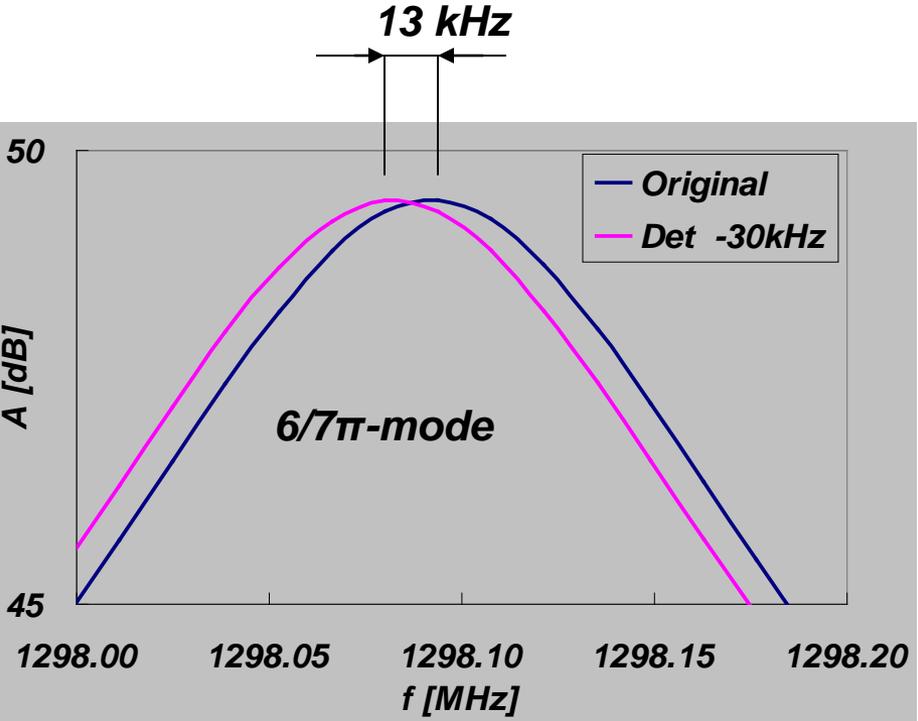
- Cavity pre-tuning after the fabrication and bulk chemical treatment
- Investigation how the field profile in cells depends on their frequency errors ( $\partial f/f < 10^{-4}$ )
- Investigation how passband frequencies depend on cell frequency errors ( $\partial f/f < 10^{-4}$ )
- Modeling of the transient state (mode beating)
- Modeling of the voltage stability during acceleration

**blue marked examples of the implementation are shown on next slides**

# 6. LEC and Transient State

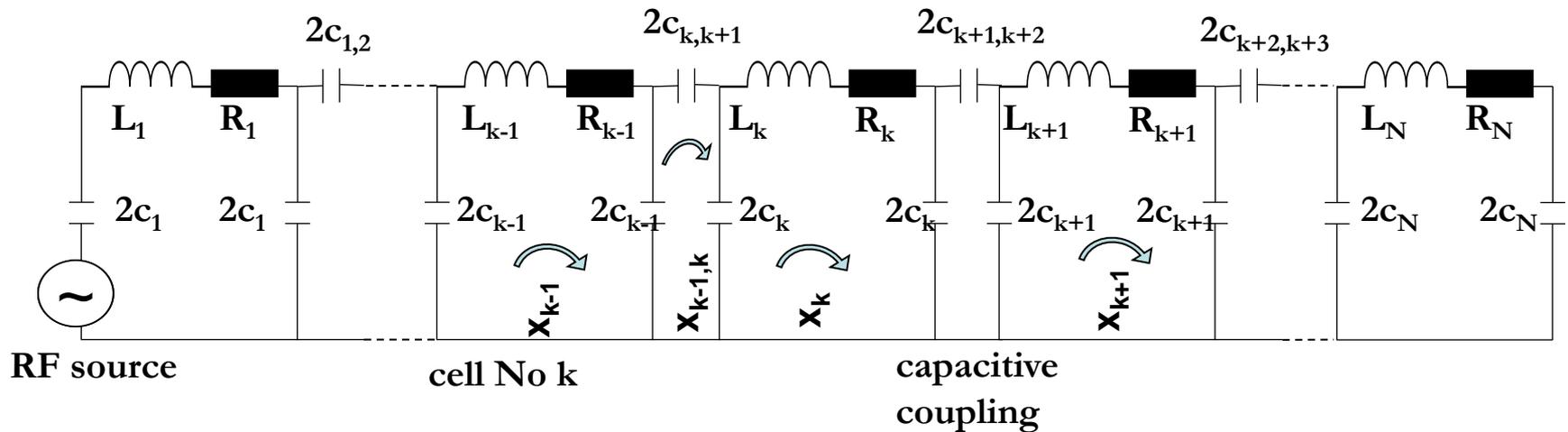
## Investigation of the FM passband frequencies sensitivity to cell frequency errors ( $\partial f/f < 10^{-4}$ )

Example: 7-cells,  $k_{cc}=1.85\%$ , 1st cell detuned by  $-30\text{kHz}$  (cell length changed  $-11\mu\text{m}$  !!, hard to model with 2D and 3D codes)



# 6. LEC and Transient State

## Transient State: Mode beating in the pulse operation



Solving the set of Kirchoff equations:

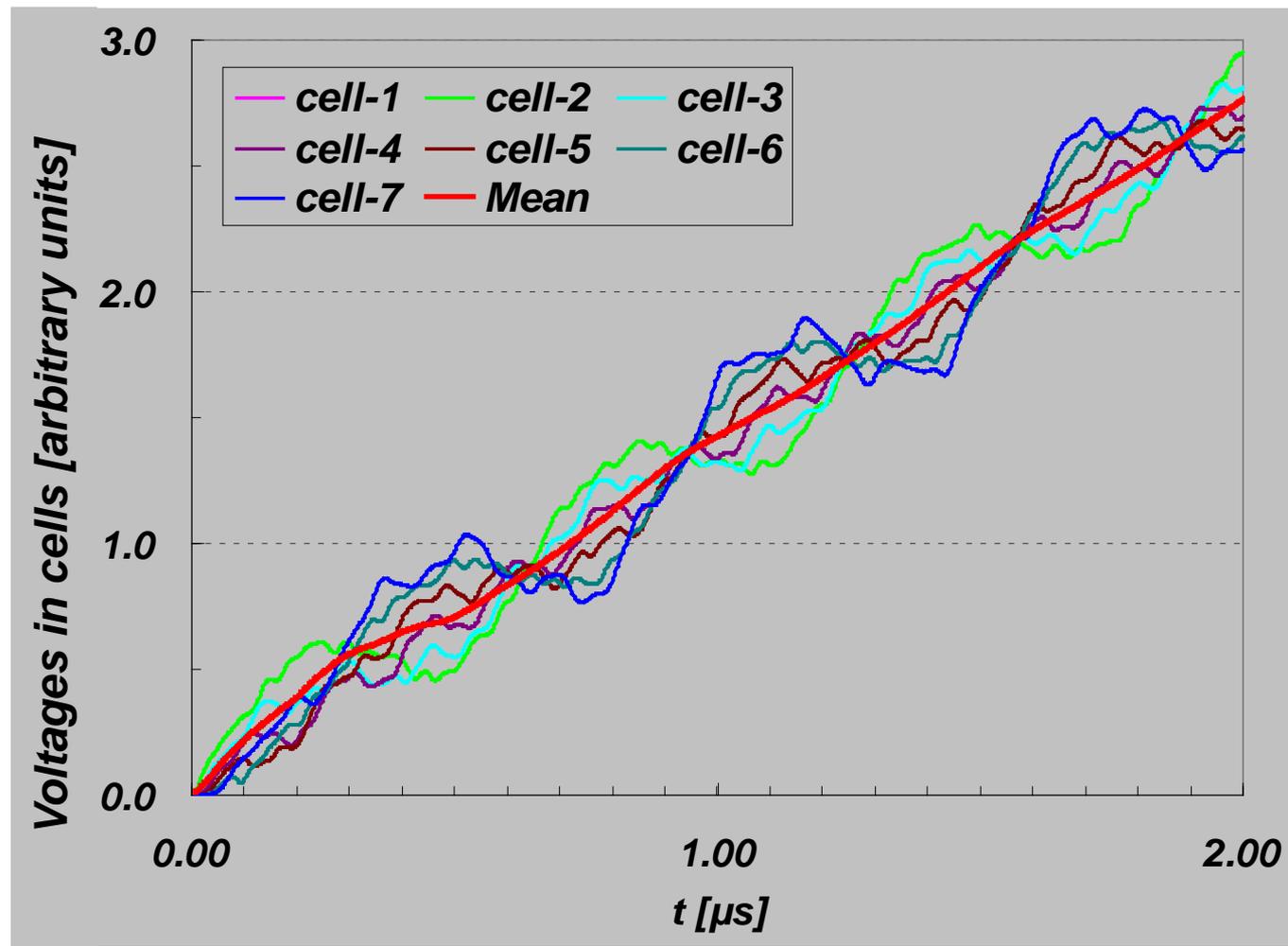
$$\begin{aligned}
 &R_1 \cdot x_1(t) + L_1 \cdot \dot{x}_1(t) + \frac{1}{C_1} \cdot \int_0^t x_1(\tau) d\tau - \frac{1}{C_1} \cdot \int_0^t x_{1,2}(\tau) d\tau = U_{-1}(t) \cdot e(t) \\
 &\dots\dots\dots \\
 &-\frac{1}{C_k} \cdot \int_0^t x_{k-1,k}(\tau) d\tau + R_k \cdot x_k(t) + L_k \cdot \dot{x}_k(t) + \frac{1}{C_k} \cdot \int_0^t x_k(\tau) d\tau - \frac{1}{C_k} \cdot \int_0^t x_{k,k+1}(\tau) d\tau = 0 \\
 &\dots\dots\dots \\
 &-\frac{1}{C_N} \cdot \int_0^t x_{N-1,N}(\tau) d\tau + R_N \cdot x_N(t) + L_N \cdot \dot{x}_N(t) + \frac{1}{C_N} \cdot \int_0^t x_N(\tau) d\tau = 0
 \end{aligned}$$

one can find voltages right after the RF-source is switched on and during the acceleration

# 6. LEC and Transient State

Modeling of the transient state (mode beating)

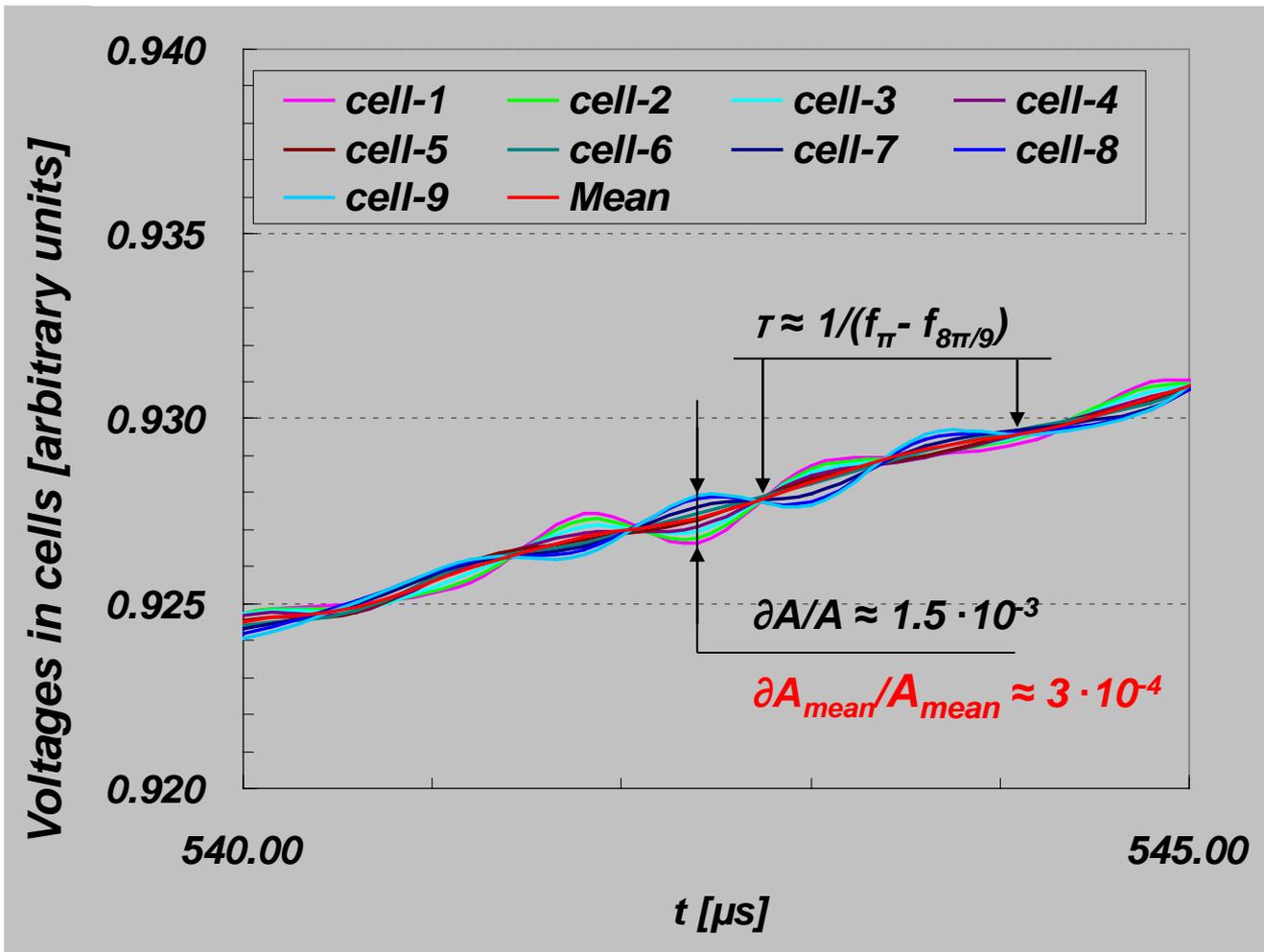
Example: 7-cells,  $k_{cc}=1.85\%$ ,  $Q_L=3.4 \cdot 10^6$



# 6. LEC and Transient State

Modeling of the transient state (mode beating at the beam arrival time)

Example: 9-cell TESLA structure,  $k_{cc}=1.85\%$ ,  $Q_L=3.8 \cdot 10^6$

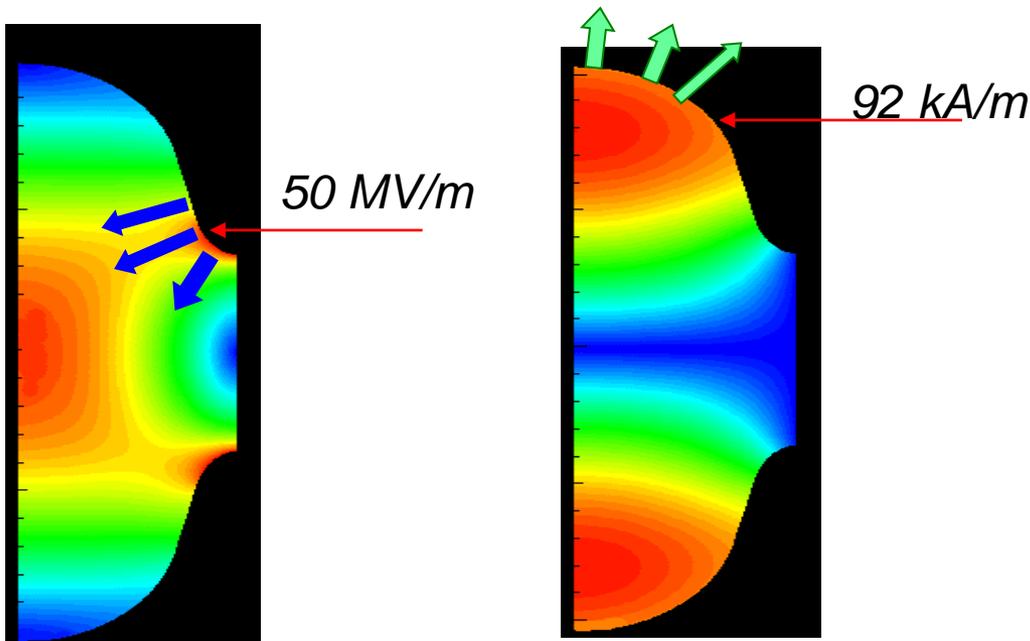


# 7. Mechanical Design

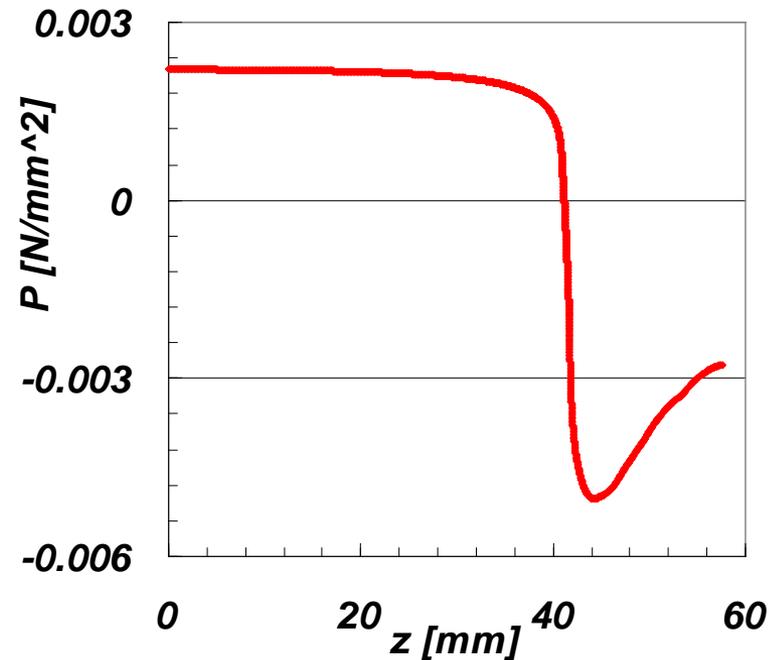
The mechanical design of a cavity follows its RF design:

- Lorentz Force Detuning
- Mechanical Resonances

Lorentz Force Detuning

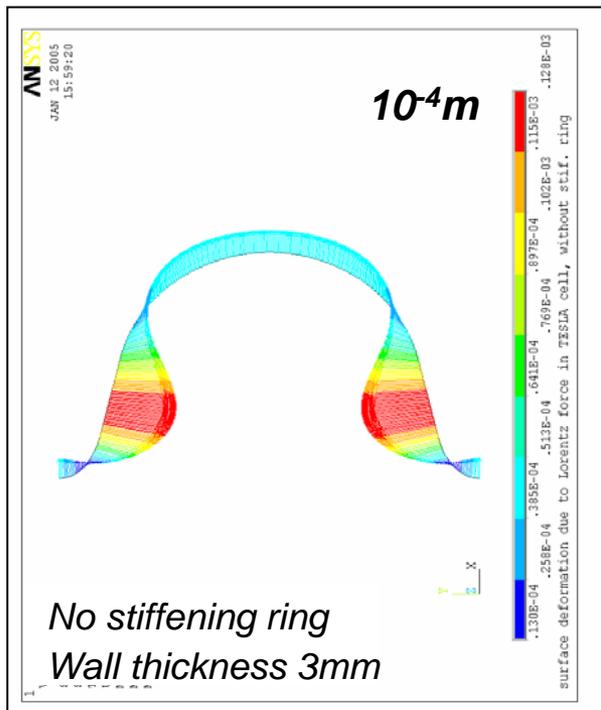
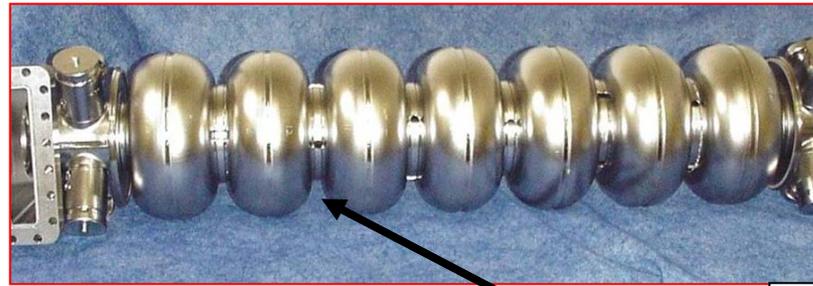


$$P = \frac{\mu_0 H_s^2 - \epsilon_0 E_s^2}{4}$$

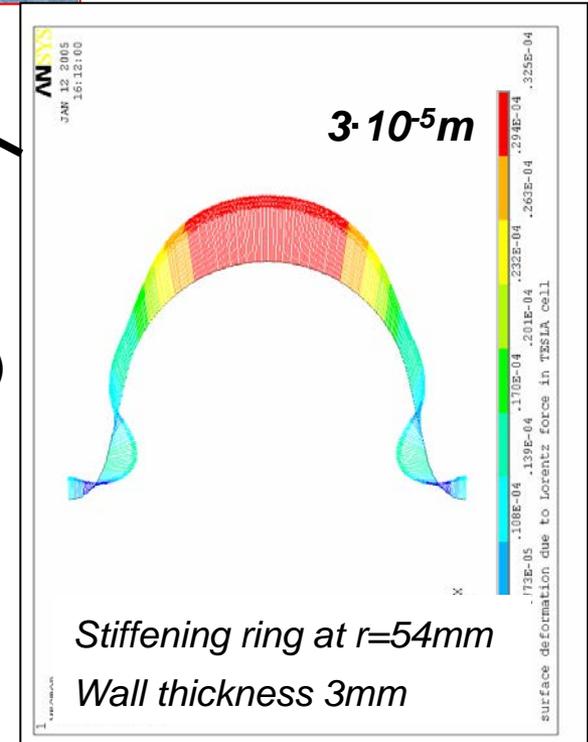


$E$  and  $H$  at  $E_{acc} = 25 \text{ MV/m}$  in the TESLA inner-cup

# 7. Mechanical Design



Surface deformation without and with stiffening ring (courtesy of I. Bonin, FERMI)



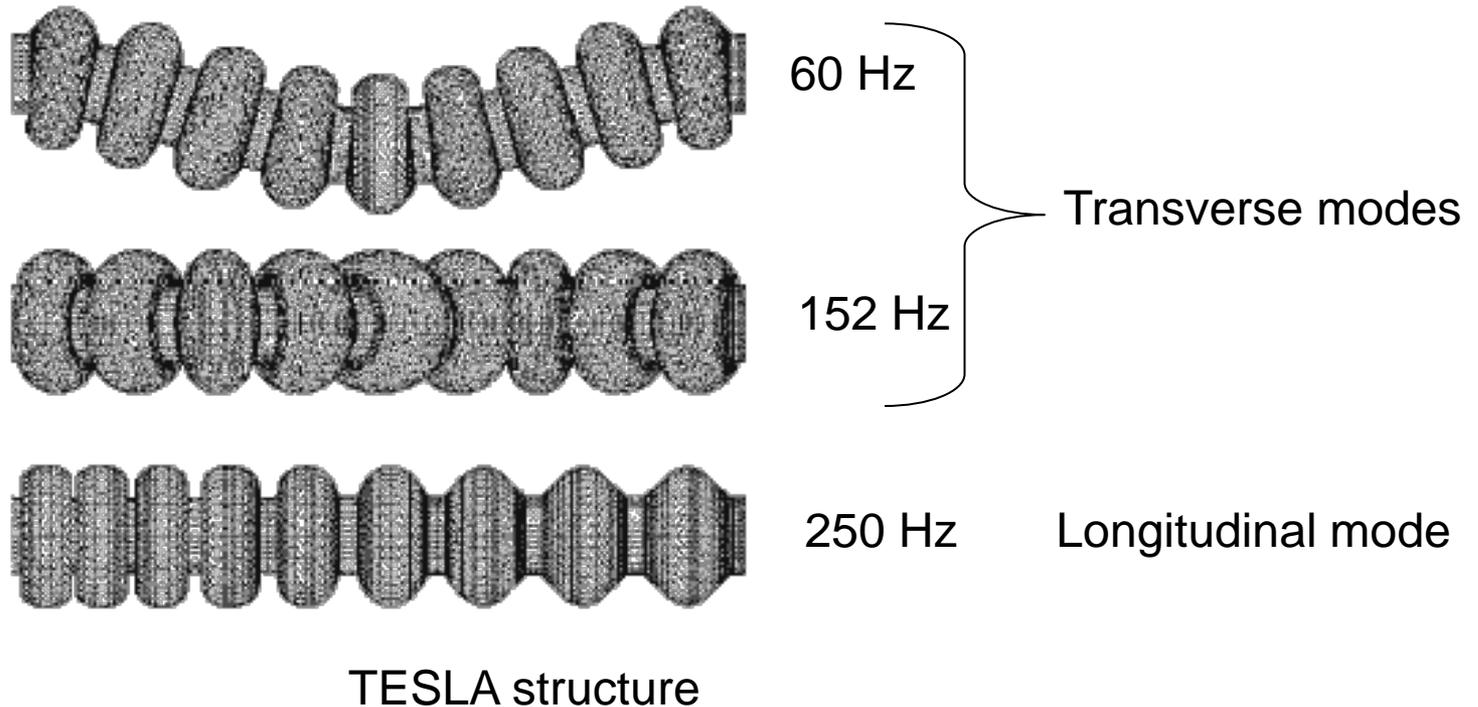
Lorentz force deformation is critical for the operation of a pulsed accelerator

$$\Delta f = k_L (E_{acc})^2$$

For TESLA cavity:  $k_L = -1 \text{ Hz}/(MV/m)^2$

# 7. Mechanical Design

## Mechanical Resonances of a multi-cell cavity

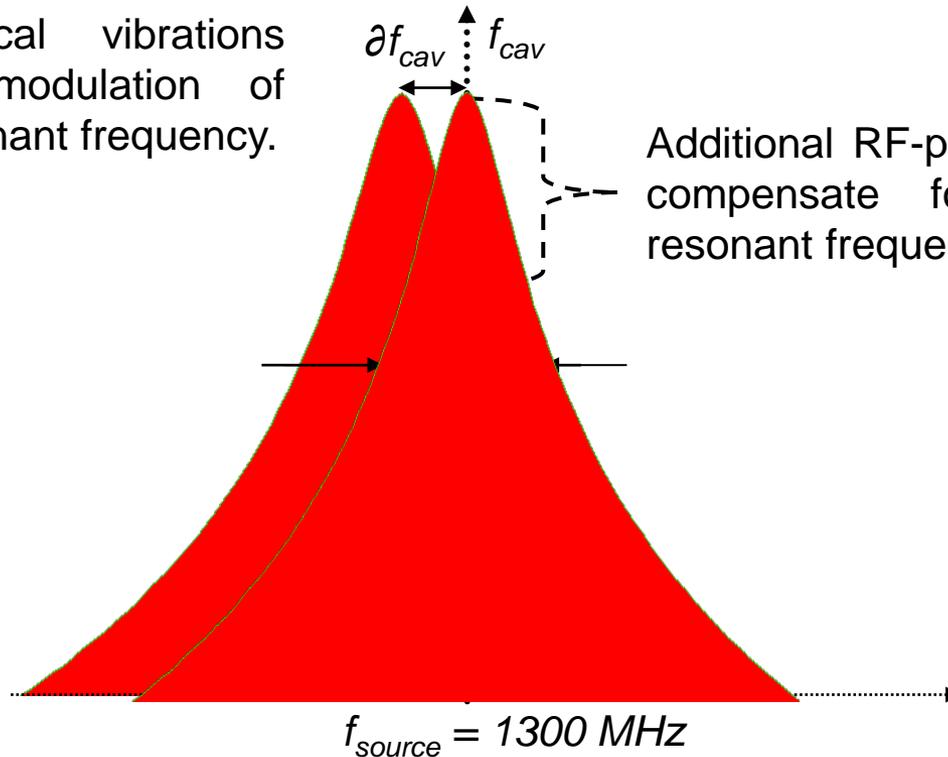


The mechanical resonances modulate frequency of the accelerating mode.  
Sources of their excitation: vacuum pumps, ground vibrations, helium pressure...

# 7. Mechanical Design

$$\Delta f_{3db} \equiv \frac{1.3 \cdot 10^9 \text{ Hz}}{2 \cdot 10^7} = 65 \text{ Hz}$$

Mechanical vibrations cause modulation of the resonant frequency.



## 8. Final Remarks and References

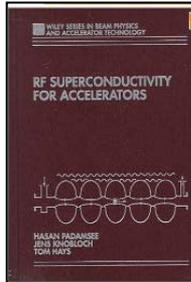
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Not discussed topics:

1. HOM dampers: loop couplers, waveguide couplers, beam line absorbers
2. Input couplers design
3. Cold frequency tuners design
4. Design of LHe vessels
5. ....

## 8. Final Remarks and References

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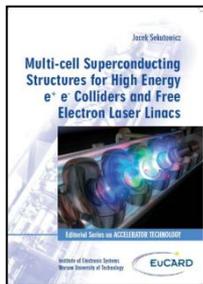


1. H. Padamsee, J. Knobloch, T. Hays, **“RF-Superconductivity for Accelerators”**, Wiley Series in Beam Physics and Accelerator Technology, 1998.

2. Proceedings of all SRF Workshops; <http://accelconf.web.cern.ch/accelconf/>



3. TESLA TDR, DESY-Report 2003



4. J.S. **“Multi-cell Superconducting Structures for High Energy  $e^+e^-$  colliders and Free electron Laser Linacs”**, CERN/FP7;  
<http://eucard.web.cern.ch/eucard/>